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Dynamic study of factors impacting on combat power

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DYNAMIC STUDY OF FACTORS IMPACTING
ON COMBAT POWER

by

Paul M. Crawford

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This thesis extends the development of the Generalized Value System (GVS), used in the Airland Advanced Researc Model (ALARM), as an on-going research effort at the Naval Postgraduate School. Specifically, the problem of determinir the multidimensional mapping of the state variables that represent the condition of an entity into the power function is ac dressed. The methods described in this thesis provide a means of aquiring this mapping function by the use of a Degrade Power Function (DPF). The DPF provides a basis for estimating the future state of an entity based on the state (conditio of the entity, virtually eliminating the exponential decay function presently in use.

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Dynamic Study of Factors Impacting on Combat Power

by

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B.S., Alabama A & M University, 1979

Submitted in partial fulfillment of the
requirements for the degree of

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from the

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March 1988

ABSTRACT

This thesis extends the development of the Generalized Value System (GVS), used in the Airland Advanced Research Model (ALARM), as an on-going research effort at the Naval Postgraduate School. Specifically, the problem of determining the multidimensional mapping of the state variables that represent the condition of an entity into the power function is addressed. The methods described in this thesis provide a means of aquiring this mapping function by the use of a Degraded Power Function (DPF). The DPF provides a basis for estimating the future state of an entity based on the state (condition) of the entity, virtually eliminating the exponential decay function presently in use.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

Modeling of the decision-making process in present combat simulations continues to be the driving factor behind the improvements in such models as the AirLand Advanced Research Model (ALARM), being developed at the Naval Postgraduate School. ALARM is designed to evaluate a U.S. Corps versus a Soviet Front. The decision-making process unquestionably impacts upon the accuracy of any combat simulation. It is therefore, very important to insure the decision-making process takes into account all factors impacting on the ultimate goal: mission accomplishment. In order to have realistic simulations, there must exist a reliable data base with all the information needed to run the simulation. Obvious shortcomings exist if the data base is **not** representative of the "real world".

To determine the combat effectiveness of a unit and make any conclusions or recommendations regarding its ability to accomplish the assigned mission requires the decision-maker to answer several questions:

- What is the assigned mission ?
- What resources are available ?
- What is the enemy situation ?
- How much time is available ?
- What is the condition of personnel ?
- What type of terrain will be encountered ?
- What are the capabilities of the friendly units ?
- What are the enemy capabilities ?

The focal point of the decision-making process is the decision-maker's appraisal of his units' combat effectiveness (ability to accomplish its assigned mission), based on the answers to the questions given above. In general, the problem is to describe the relative importance of the various resources available and decide if the unit can accomplish its assigned mission. Staff planning must take into account how the available resources constrain the combat operation. Once the decision maker has gathered both the information needed and the recommendations of his staff, he then determines the best course

of action to take in order to accomplish the assigned mission. The more information the decision-maker has at his disposal the better he can make a critical decision with reasonable certainty.

This thesis presents a dynamic approach of mapping the available resources (state variables) into the unit's power. In order to accomplish this mapping function and incorporate the state variables in the computation of power there is first a need to establish a data base which relates the state variables to combat effectiveness.

There are several means of determining a unit's effectiveness. Undoubtedly, the best method is actual combat. However, there are other methods, short of warfare, which provide a viable determination of combat effectiveness. One of the better methods is based on the judgement of expert decision-makers.

The primary goal of this thesis is the development of a usable and realistic model which will describe the relationship between the state variables, representing the resources available to a combat unit, and the resulting combat effectiveness of that unit and the unit's ability to accomplish its assigned combat mission. It is important to note that the basic methodology outlined in this paper is applicable to a wide variety of situations in which the decision-maker is required to view the current situation and make an assessment of the unit's combat effectiveness.

Background information is presented in Chapter II, describing the techniques and systems being used in ALARM. Chapter III discusses different techniques of gathering information from judges, along with a detailed explanation of the technique used in this thesis. A substantial effort was made to determine the best techniques of asking the right questions and the method chosen was deemed the best of the methods investigated. In Chapter IV the data is analyzed and the best fitting model is discussed, along with an example of the sensitivity of the model developed. Finally, future applications are suggested, based on the methodology developed in this thesis.

II. BACKGROUND

In present day military operations there has emerged a term called Airland battle, which is becoming the doctrine of the U.S. Army. This doctrine states that battles will not be limited to the forward fighting elements but will involve the total battle zone to include the rear areas. In order to more closely examine this doctrine and its effects on battle outcomes, computer simulations are used to examine the effects of this doctrine. This need for evaluation motivated the design of the AirLand Advanced Research Model (ALARM). ALARM, under development at the Naval Postgraduate School since January 1985, focuses on investigations of the effectiveness of the Airland battle doctrine for a U.S. Corps battle. [Ref. 1] ALARM is capable of man-in-the-loop decisions but is primarily being developed as a systemic (i.e., no man-in-the-loop) U.S. Corps versus Soviet Front model. The ALARM concept incorporates three basic methodologies: Time Domain Networks, Cartesian Space Networks, and Generalized Value System.

The function of time domain networks is "to handle the planning function or activity within the ALARM system. The time domain network consists of nodes and arcs (or links) connecting the nodes. The key to time domain networks is that arcs do not represent distance, but represent the passage of time or the completion of a sub-activity which leads to the completion of the entire activity represented by the network." [Ref. 1: pp.6-8]

Cartesian space networks consist of "a series of networks each representing a different aspect of the battlefield. Each network represents physical connections between points on the battlefield. ALARM will have three or more Cartesian space networks." [Ref. 1: p.8] The three networks that have been identified are as follows:

1. Terrain and Transportation networks [Ref. 2]
2. Communications network
3. Logistics Resupply network

The Generalized Value System (GVS), an axiomatic value system used as a tool for evaluating the power of entities at present and future times, was first proposed in 1985 by Professor Schoenstadt of the Naval Postgraduate School [Ref. 3] and was expanded

in 1986 by Robert Kilmer [Ref. 4]. This procedure provides a means of forecasting future states of entities in continuous time, allows different entities to be compared, and works for all entities on the battlefield. Essentially, "GVS is a procedure for quantifying the capabilities and importance of entities on the battlefield at some future time $(t+x)$. It does this by using algorithms to predict future entity or situation states at time $(t+x)$ based on the situation at time (t). Thus GVS provides the framework for forcasting future states of entities in continuous time." [Ref. 1: p.8]

Inherent in this procedure is the concept of unit effectiveness and assigning that unit (entity) a value which is consistent with other units (entities) on the battlefield. This value, called "Power," is the value that GVS determines based on the usefulness of an entity at that time.

A. POWER

Power is a subjective value given to an element or unit on the battlefield. In GVS it combines both inherent and derived power. This value is then used as a planning factor in the combat simulation. Accuracy in assigning values to elements is crucial to the reliability of this methodology. Hence, a great deal of time and effort has been spent in the assignment of such values. Several of the techniques used in determining values are firepower scores, user input values, and Generalized Value System (GVS). Firepower scores use an aggregated approach to determine the value of an entity (i.e., the value of an entity depends on the additive value of all of its weapon systems). User Input Values are assigned along with priorities of target selection by the user. Generalized Value System (GVS) assigns a value to different entities on the battlefield, to include non-combat entities such as combat support units, bridges, etc.. Each value is comparable to other values. In GVS an entity relates to a unit or even a bridge and can have two types of power: inherent and/or derived. A combat unit will normally have a combination of inherent and derived power, while a combat support unit may only have derived power.

1. Inherent power

Inherent power is the ability of a unit (entity) to affect the outcome of a battle by the disruption, destruction, and confusion of the enemy forces. It also includes the effectiveness and survivability of a unit. Kilmer defines inherent power as "the ability to directly affect the states of enemy entities or of entities that the enemy is using or planning to use (e.g., a bridge)." [Ref. 4: p.32] Inherent power types that need to be defined in order to focus the direction of this paper are Basic Inherent Power (BIP) and

Situational Inherent Power (SIP). Basic inherent power (BIP) is the inherent power that a unit possesses when it is at full strength, in position and ready to perform its most likely mission. Situational Inherent Power (SIP) is the inherent power that a unit is predicted to have at some future time, t.

To calculate the Situational Inherent Power (SIP) of an entity, the time the entity is expected to be ready to accomplish its mission is estimated from the time domain networks. This time is called the available time (t_A). Other aspects that must be determined are the rate of attaining readiness (D), the present time (t_p), and the Predicted Adjusted Basic Inherent Power (PABIP). Once these values are known a unit's SIP can be computed using equation 2.1:

$$SIP(t \mid t_p) = PABIP(\vec{SX}1(t_p)) \times e^{-D(t_A - t)} \quad for \ 0 \le t \le t_A , \tag{2.1}$$

$$SIP(t \mid t_p) = PABIP(t \mid t_p) \quad for \ t \ge t_A ,$$

where:

D is the rate at which an entity is attaining readiness,

t_A = Time the entity will be available to execute the assigned mission,

t_p = Present time, and

$\vec{SX}1(t)$ = State (condition) of entity X1 at time t expressed as a vector of state values.

It can be seen that the SIP of an entity is dependent on the PABIP, but the PABIP is dependent on the Adjusted Basic Inherent Power (ABIP). The ABIP is the percent degradation at any time, t. Therefore ABIP = (percent degradataion) × BIP. From this the PABIP can be calculated using equation 2.2:

$$PABIP(X1(t) | \vec{SX}1(t_p)) = ABIP(\vec{SX}1(t_p)) \times e^{-L(t - t_p)} \quad for \ t_p \le t \le t_E , \tag{2.2}$$

where:

L is a rate of change which may have several components such as attrition, reenforcements, and logistics effects, and

t_E is the end of the planning horizon.

This equation uses an exponential decay function to calculate PABIP of an entity over the entire planning horizon. Note that, in general, t_A may occur either before or after

t_E . This thesis proposes a method of determining the PABIP based on the state of an entity and not on an exponential decay function. This method will be discussed in detail in Chapter III.

2. Derived Power

Derived power consists of the power a unit has when it supports another unit or provides the services needed by a unit in accomplishing its assigned mission. Kilmer defines derived power of an entity as "the power it possesses because of its ability to influence the states of other friendly entities or of entities that its forces are planning to use." [Ref. 4: p.33]

The estimation of power in modern computer wargaming exercises has been evaluated and questioned for many years. There is a great need to have the best representation of a units' power. In the present ALARM the power is based on the value GVS gives the entity and GVS uses an exponential decay function which is not directly related to the present state of an entity. ALARM does have formulations to determine the units' present strengths and thus the the state of the unit (entity). The state of the entity takes into account the factors impacting on the ability of the entity to accomplish its assigned mission.

B. THESIS MOTIVATION

In order to make combat simulations more accurate and have less arbitrary assignment of values for power, there is a need for a multivariate mapping function of state vectors to unit power. This mapping function will allow a future state decision process to replace the exponential function presently being used in determining the PABIP. Conversely, this mapping function will allow GVS to determine the future state of an entity based on the state variables which are representative of the entity. This mapping function is discussed in Chapter III and is based on the collective judgement of a large group of decision-makers vice an arbitrary assignment of values. The combined experience of the decision-makers to determine the future state of an entity through the mapping function creates a more accurate combat simulation and more accurate results.

Power, therefore, should not be an arbitrary determination made by one individual or a small group of "experts," but should be decided by a large group of actual decision-makers. Since there is no data base established from which to determine the power of an entity, there exists a need to establish such a data base. This need to more

accurately represent the actual battlefield by developing a multivariate mapping function of state variables into unit power. This will enable a better determination of the future state of an entity and make GVS a more usable technique. This is the motivation for this thesis.

III. THE EXPERIMENT

As stated in Chapter I, the primary motivation for this thesis is to develop a multivariate mapping function of state variables into unit power in order to better determine the future state of an entity. In conducting an experiment to develop this function it was first necessary to determine a technique that would allow for the most precise, reliable responses from a select group of judges. First a discussion of the determination of combat effectiveness by a decision-maker is needed.

To determine the combat effectiveness of a unit and draw any conclusions regarding whether or not the unit can accomplish its assigned mission requires the decision-maker to look at several questions:

- What is the assigned mission ?
- What resources are available ?
- What is the enemy situation ?
- How much time is available ?
- What is the condition of personnel ?
- What type of terrain is being defended ?
- What are the capabilities of the unit ?
- What are the enemy capabilities ?

As pointed out in Chapter I, the focal point of the decision-making process is the decision makers' appraisal of his units' combat effectiveness (ability to accomplish its assigned mission) based on the answers to the questions listed above. The capabilities of the friendly units are critical in this planning process (available resources can determine the feasibility of any defensive operation). Once the decision maker has gathered both the needed information and the recommendations of his staff, he then determines the best course of action to take in order to accomplish the assigned mission. The more information the decision-maker has at his disposal, the better he can make a critical decision with reasonable certainty.

The questions posed above are a key element in the decision-making process and as such they are the descriptors of the unit (i.e., they tell the decision maker the STATE of his unit). These descriptors will be called the state variables for this study and the

STATE of the unit will be based on the value of each variable. In order to better understand this concept a thorough description of the system and state variables is required. It is important to provide as detailed and comprehensive a list of variables impacting on combat effectiveness as feasible, so that any simplifying assumptions and state variable omissions made during the course of problem definition and analysis may be directly related to the initial system description.

In general the problem is to describe the relative importance of the various state variables in a subjective decision process. It is hypothesized that the decision maker considers his units' capabilities and his vulnerabilities in deciding on his units' ability to accomplish its mission. There are 7 variables that are considered, in this study, to affect combat outcome.

1. Personnel strength (foxhole strength).
2. Available ammunition.
3. Key operational weapon systems.
4. Fuel and oil products available (POL).
5. Chain of command (key leaders).
6. Level of experience of unit and leaders.
7. Enemy force capabilities.

It would be ideal to be able to develop this model utilizing all of these variables and evaluate them all at many levels, but that would clearly be unmanageable. We will therefore reduce the number of these variables to a manageable level. The mission of the unit will be limited to deliberate defense. The chain of command will not be evaluated since when evaluating personnel strength it is assumed that this will include key leaders. As experience of the unit and of the leaders is not quantifiable, it will therefore, not be evaluated. Enemy forces will be a Motorized rifle regiment, and therefore will not change. This leaves four variables which will be used in this model:

1. Current personnel strength, as a percent of the authorized Table of Organization and Equipment (TO&E), (The TO&E is a document that reflects all of the types and quantities of personnel and equipment a particular unit is authorized),
2. Current ammunition available, as a percent of the authorized amount,
3. Current combat vehicles operational, as a percent of the authorized TO&E,
4. Current POL fuel and oil products available, as a percent of the authorized amount.

It must now be determined at what levels these variables will be evaluated. Based on discussions with combat arms officers and other experts, as well as review of an earlier research paper by Elizabeth Etheridge and Michael Anderson [Ref. 5], it was determined that personnel and combat vehicles would be evaluated at three levels (100%, 75%, 50%) and ammunition and POL would be evaluated at four levels (100%, 75%, 50%, 25%), constituting a factoral design. A factoral experiment is one in which all levels of a given variable are combined with all levels of every other variable in the experiment. A 4^k factoral experiment is a case where k factors are evaluated each at 4 levels. It can easily be seen that a 4^4 factoral experiment requires 4^4 = 256 separate unique combinations, each representing a different possible value of combat effectiveness. In the case of the model being developed where two variables are being evaluated at three levels and two variables are being evaluated at four levels, there exists a $(3^2) \times (4^2)$ factoral experiment requiring 144 separate evaluations.

A. STATEMENT OF THE PROBLEM

Determining the relative importance of the various system state variables in a decision-maker's estimate of particular variables relative to his units' ability to accomplish its assigned mission is the ultimate objective of this study. In determining the relative value of these variables, a subjective evaluation was used in order to establish a data base. Several techniques were considered in the establishment of this data base: a continuous response scale, the paired comparison test, ordinal judgements and categorical judgements [Ref. 6]. Each is discussed briefly below.

B. CONTINUOUS RESPONSE SCALE

A continuous response scale allows the judges to respond on a continuous scale from 0 to 100, based on the judges' feelings of the items he is evaluating. This type of test could lead to an Analysis of Variance (ANOVA) design which is commonly used to determine interrelationships. The problem with this design is that judges may find it difficult to make a judgement on a continuous scale from 0 to 100 and for this reason this method was rejected.

C. PAIRED COMPARISON TEST

In the paired comparison test, the judges are presented with pairs of items being evaluated (instances) and are then asked to determine which possesses the greater value. For a paired comparison test the judges are asked to compare each of the possible pairs, which involves $n(n-1)/2$ comparisons. For this study it would involve $144(144-1)/2$ =

10,296 comparisons. This test allows each variable to be compared with all other variables independently helping to determine any interrelationships between variables. The problem with this method, however, is that it requires a large number of comparisons. Since the variables are paired together, a higher order interrelationship cannot be determined. Therefore, it was not used in this study.

D. ORDINAL JUDGEMENTS

In making ordinal judgements, judges are asked to rank instances based on their "feelings" about an instance and subsequently set up a rank for all of the instances that they are evaluating. The problem with this technique is that a judge may not be able to discriminate between two instances in terms of the value he may feel they possess, and when comparing a large number of instances the problem is confounded. Also, some judges may not rank all instances. This technique was not used since the number of instances each judge would have to rank would be prohibitive.

E. CATEGORICAL JUDGEMENTS

This method requires the judges to select the category that they think best represents an instance. The categories are assumed to be a mutually exclusive set of successive intervals on the variable's scale. There are descriptors with each category that help the judges with their task, (i.e., strongly agree, agree, no opinion, disagree, strongly disagree). Normality is an assumption of this technique, where the judges feelings about the scale value of unit effectiveness for an instance is normally distributed, as seen in Figure 1.

The categorical judgement technique did not require an exhaustive amount of work by the judges and discussions with several experts in the field at the Naval Postgraduate School provided greater assurance that choosing this technique would result in the final development of a usable model.

F. CONSTRUCTING INTERVAL SCALES FROM CATEGORICAL RESPONSES

Using the categorical judgements technique to transform the judgemental responses to an interval scale was determined to be the best technique. Hence finding an interval scale based on the responses of the judges requires ten steps. [Ref. 7: p.6] These steps are summorized here and given in detail, in Section G below. This technique also requires 4 assumptions.

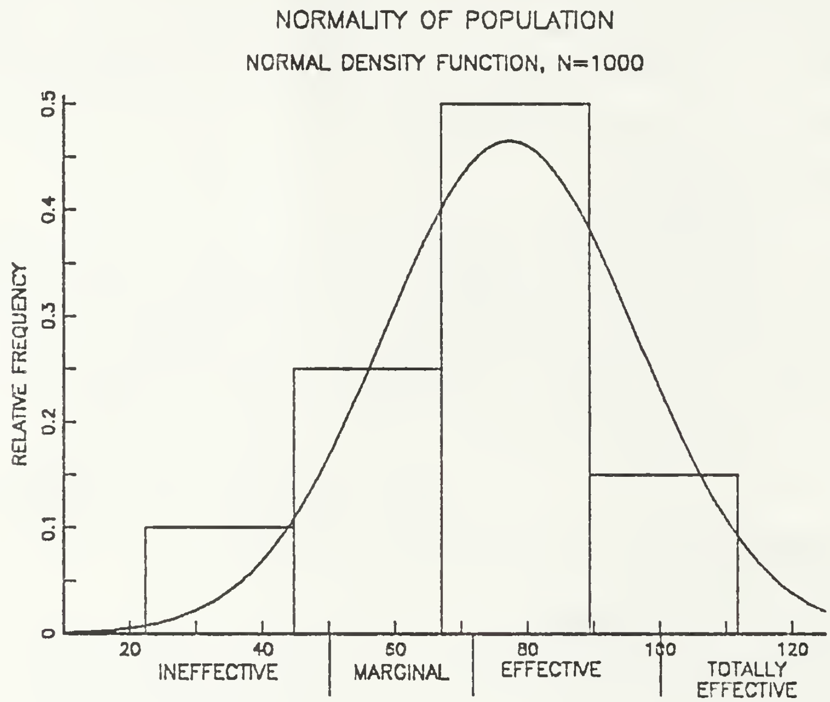


Figure 1. Normal Population of Judges Feelings of Unit Effectiveness

1. The judges' "feelings" about the scale value of scenario (instance) i are assumed to be a normally distributed random variable with mean μ_i and variance σ_i^2.
2. Categories are a mutually exclusive set of of successive intervals which collectively exhaust the continuum of the scale.
3. The judges feelings about a category's upper bound as a normally distributed random variable, so for category j the upper bound would be a normally distributed random variable with mean μ_j and variance σ_j^2 .
4. All category bounds are assumed to have the same variance, so for all j, $\sigma_j^2 = c$.

The responses are recorded in a table that lists all of the raw frequencies for each instance. The raw frequencies are then divided by the total number of judges who answered that instance to determine relative frequencies. The relative frequencies are then

used to calculate the cumulative relative frequencies which are likened to the cumulative frequencies under a normal curve. The cumulative relative frequencies constitute the P array, where P = $\| p_{ij} \|$, and p_{ij} is the cumulative relative frequency for instance i, and category j. These p_{ij} are then grouped with other instances having similar category values (i.e., instances with all values in the categories A, B, and C are grouped with all other instances with values in categories A, B, and C). Once the instances are grouped, all values of $p_{ij} \geq 0.98$ and $p_{ij} \leq 0.02$ are removed to avoid any undue influence by a small number of judges. Treating these values of p_{ij} as leftward areas under a Normal (0,1) curve, a table of the Normal CDF values is developed. These values are the Z array used in the computations, where Z = $\| z_{ij} \|$, and z_{ij} is the value from the normal table for instance i, and category j. The row, column and grand average are calculated from this Z array, along with the estimate of the standard deviation. The scale values of the instances are then computed using the values derived:

Scale Values = Grand Average - Row Average × Standard Deviation.

These scale values are then transformed using a linear transformation.

G. STEP BY STEP INTERVAL SCALE DEVELOPMENT

The development of the interval scale as presented in [Ref. 7] is outlined below. For the present it is assumed that all p_{ij} are such that $0.02 \leq p_{ij} \leq 0.98$, so that the Z array is complete. The array P is defined as the cumulative frequencies of each instance and z_{ij} is defined as the values determined from a table of the normal distribution for the p_{ij} values.

1. Arrange the raw frequency data in a table where the rows are scenarios (instances) and the columns are categories. Columns should be in rank order with the least favorable category in the left column and the most favorable in the right column.

2. Compute the relative cumulative frequencies for each row and record these values in a new table. This table is referred to as the P array and all values of $p_{ij} > 0.98$ and $p_{ij} < 0.02$ are removed. This creates an $n \times (m-k)$ array, where k is the number of columns removed.

3. Treating these p_{ij} values as leftward areas under a Normal (0,1) curve, look up the values of Z from a table of the normal distribution. Record these as a new table which will be the Z = $\| z_{ij} \|$ array for the computations that follow.

4. For each instance, i, in the Z array, compute the row average, $\bar{z}_i$

5. For each column j in the Z array, compute the column average, b_j . Note that b_j is the value of the upper bound of category j on the scale being developed.

6. Compute the grand average, $\bar{b}$, of all values of the Z array.

7. Compute B = $\sum_{j=1}^{m-k} (b_j - \bar{b})^2$, the sum of squared column differences.

8. For each row compute $A_i = \sum_{j=1}^{m-k} (z_{ij} - \bar{z}_i)^2$, the sum of squared individual differences.

9. For each scenario compute $\sqrt{\frac{B}{A_i}}$, an estimate of $\sqrt{\sigma_i^2 + c}$.

10. Finally for each row (scenario) compute $S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$, for all i.

These S_i 's are the scale values of the instances, on the same interval scale as the category bounds, b_j . Now all instances and category bounds are on the desired scale and any linear transformation $Y = \alpha + \beta \times X, \ \beta > 0$, may now be performed to adjust the scale as desired, using the same transformation to move both scenarios and category bounds. [Ref. 7: pp.14-15.]

In order to better understand this concept a short example will be presented. Raw data is first gathered and set up in matrix form showing the frequency that a category was chosen. The least desirable category appears in the leftmost column. For this example it is column A, and for the study conducted the letters correspond to the following:

A = Totally Ineffective,

B = Ineffective,

C = Marginal,

D = Effective,

E = Totally Effective.

The first part of step 1 above is to record the raw frequencies (Table 1). The numbers in the left column correspond to actual numbers assigned to an instance of the pilot study. The pilot study is discussed in Section II below.

Table 1. RAW FREQUENCIES

No.	A	B	C	D	E
52	0	1	3	11	5
58	0	1	4	14	1
66	0	1	14	4	1
71	0	1	13	5	1
92	0	1	13	5	1

The second part of step 1 is to divide the frequencies by the total number of judges (20) who answered the questions to determine the relative frequencies (Table 2).

Table 2. RELATIVE FREQUENCIES

No.	A	B	C	D	E
52	0.00	0.05	0.15	0.55	0.25
58	0.00	0.05	0.2	0.7	0.05
66	0.00	0.05	0.7	0.2	0.05
71	0.00	0.05	0.65	0.25	0.05
92	0.00	0.05	0.65	0.25	0.05

Step 2 involves determining the cumulative relative frequency of the array by summing each column with the values of the columns to the left. This table is called the P array and is given in Table 3.

Table 3. CUMULATIVE RELATIVE FREQUENCIES

No.	A	B	C	D	E
52	0	0.05	0.2	0.75	1
58	0	0.05	0.25	0.95	1
66	0	0.05	0.75	0.95	1
71	0	0.05	0.7	0.95	1
92	0	0.05	0.7	0.95	1

The second part of step 2 is to remove all values of $p_{ij} > 0.98$ and $p_{ij} < 0.02$. Therefore, the column of 0's and the column of 1's is removed from Table 3, resulting in Table 4.

Table 4. REMOVE ZEROS AND ONES

No.	B	C	D
52	0.05	0.2	0.75
58	0.05	0.25	0.95
66	0.05	0.75	0.95
71	0.05	0.7	0.95
92	0.05	0.7	0.95

In step 3 the p_{ij}'s are treated as leftward areas under a Normal (0,1) curve. The values of Z, taken from the table of the standard normal distribution, are recorded as a new table of the Z array (Table 5).

Table 5. NORMALIZED VALUES

No.	B	C	D
52	-1.64521	-0.841457	0.674189
58	-1.64521	-0.674189	1.64521
66	-1.64521	0.674189	1.64521
71	-1.64521	0.524002	1.64521
92	-1.64521	0.524002	1.64521

Step 4 involves computing the row average, $\bar{z}_i$, for each scenario, i.

In step 5, for each category j, compute the column average, b_j , and note that b_j is the value of the upper bound of category j on the scale.

Step 6 consists of computing the grand average $\bar{b}$. The results of these three steps are shown in Table 6.

Table 6. NORMALIZED TABLE

	z_{ij}			$\bar{z}_i = \sum \frac{z_i}{(m-k)}$
No.	B	C	D	
52	-1.64521	-0.841457	0.674189	-0.604159
58	-1.64521	-0.674189	1.64521	-0.224729
66	-1.64521	0.674189	1.64521	0.224729
71	-1.64521	0.524002	1.64521	0.174667
92	-1.64521	0.524002	1.64521	0.174667
Col Sum	-8.22605	0.206547	7.25503	Grand Average
b_j	-1.64521	0.041309	1.45101	$\bar{b} = -0.050965$

In step 7, B = $\sum_{j=1}^{m-k}(b_j - \bar{b})^2$, the sum of square column differences is computed as follows:

$B = (-1.64521 - (-0.050965))^2 + (0.041309 - (-0.050965))^2 + (1.45101 - (-0.050965))^2$,

B = 4.806061 .

In step 8 compute $A_i = \sum_{j=1}^{m-k} (z_{ij} - \bar{z}_i)^2$, the sum of square individual differences (Table 7).

Table 7. SUM OF SQUARE DIFFERENCES

$A_{ij} = (z_{ij} - \bar{z}_i)^2$				$A_i = \sum A_{ij}$
No.	B	C	D	
52	1.08379	0.0563099	1.63418	2.774274
58	2.01777	0.202014	3.49668	5.716464
66	3.49668	0.202014	2.01777	5.716464
71	3.31196	0.122035	2.1625	5.596493
92	3.31196	0.122035	2.1625	5.596493

In step 9 an estimate of the standard deviation $\sqrt{\sigma_i^2 + c}$ is computed by $\sqrt{\frac{B}{A_i}}$.

Finally in step 10, for each row (scenario) compute $S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$, for all i (Table 8).

Table 8. SCALE VALUES OF INSTANCES

No.	$S_i = \bar{b} - \bar{Z}_i \times \sqrt{\frac{B}{A_i}}$
52	$0.744226 = -0.050965 - (-0.604159 \times \sqrt{\frac{4.806056}{2.77427}})$
58	$0.155094 = -0.050965 - (-0.224729 \times \sqrt{\frac{4.806056}{5.71646}})$
66	$-0.257024 = -0.050965 - (0.224729 \times \sqrt{\frac{4.806056}{5.71646}})$
71	$-0.212828 = -0.050965 - (0.174667 \times \sqrt{\frac{4.806056}{5.59649}})$
92	$-0.212828 = -0.050965 - (0.174667 \times \sqrt{\frac{4.806056}{5.59649}})$

The column averages that correspond to the upper bound of the instances are given in Table 9.

Table 9. COLUMN AVERAGES

B	C	D
-1.64521	0.0413094	1.45101

A linear transformation $Y = \alpha + \beta \times X,\ \ \beta > 0$, may now be performed to adjust the scale as desired, using the same transformation to move both scenarios and category bounds. In this example the transformation of the S_i's was done by arbitrarily assigning a value of 100 as the upper bound of category D and zero (0) as the upper bound on category B. This results in the solving of two simultaneous equations with two unknowns. These two simultaneous equations are:

$$100 = \alpha + \beta \times (1.45101),$$

$$0 = \alpha + \beta \times (-1.64521).$$

Utilizing this transformation results in the values given in Table 10.

Table 10. TRANSFORMED VALUES

B	C	D
0	54.47	100

No.	Transformed Values
52	77.173
58	58.145
66	44.835
71	46.262
92	46.262

The transformed values of both the category upper bounds and the scale values of instances are now all on one interval scale as seen in Fig. 2. This completes the transformation of categorical responses to an interval scale.

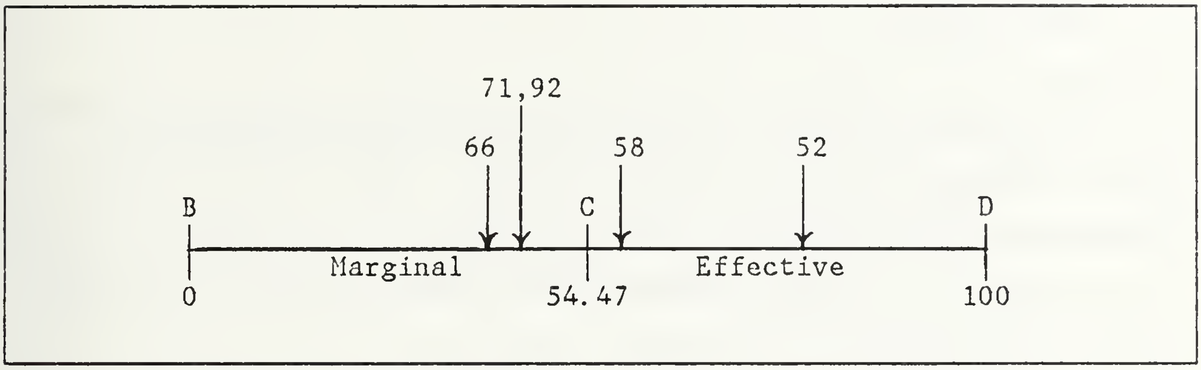


Figure 2. Transformed Values on Interval Scale

II. PILOT STUDY

The judgemental data base in this study involved the design of a (PILOT) questionnaire and its distribution to a representative group of Army officers at the Naval Postgraduate School. These Army officers will be referred to as "judges" in this study. The pilot study is used to determine the reasonableness and validity of the data gathering techniques. The study also allows changes to be made to correct any problems that may arise before sending the questionnaire out Army wide. The judges were asked to assume the role of the decision-maker of a Mechanized Infantry battalion with one mission: deliberate defense. It also provided the judges with the present STATE of the unit and the situation. The present STATE was reflected as a percentage of personnel, ammunition, combat vehicles and POL on hand or available. Two of these indicators (Personnel and Combat Vehicles) were evaluated at three levels (100%, 75%, 50%), and two (Ammunition and POL) were evaluated at four levels (100%, 75%, 50%, 25%). This approach required $(3^2) \times (4^2) = 144$ unique scenarios, each representing a different unit capability.

In this pilot study 75 questionnaires were sent to U.S. Army officers (judges) with each questionnaire containing 48 different scenarios. A sample questionnaire is presented in Appendix A. The judges were asked to determine, based on the situation and his units' present STATE, to what degree his unit could accomplish its assigned mission. From each of the scenarios the judges determine, based on the STATE of his unit, the ability of the unit to accomplish its assigned mission. This ability was then rated in one of the following categories:

- Totally Ineffective,
- Ineffective,
- Marginally Effective,

- Effective,
- Totally Effective.

In addition to the questions on effectiveness the judges were asked career questions, (e.g., branch of service, rank, staff experience), to determine the experience levels of the judges. The judges were also asked if they considered 50% for personnel and combat vehicles to be a lower bound for a unit being capable of accomplishing its mission. These responses are presented in Appendix B, as a guide for future studies of this type. Of the 75 questionnaires 60 were returned and 15 were not returned until after the analysis had been completed. The analysis was conducted and the model constructed based on the first 60 questionnaires returned.

In general, the questionnaires required approximately 45 minutes to 1 hour to complete. Judges were asked to determine at what level their unit could accomplish its assigned combat mission based on the various combat situations presented. The experiment consisted of 144 different scenarios, and since the number of Army officers at the Naval Postgraduate School was limited, the experiment was organized into three groups of 48 scenarios each. The scenarios that each group received and the order of the scenarios in each group were randomly determined with the use of a random number generator. This was done to avoid a trend in any given group, since it is undesirable for one group to have a predominance of variables with high or low percentages. Each scenario had the same mission, enemy force and situation while the four independent variables were varied. In so doing, the relationships of the independent variables and best fitting model was determined as discussed in Chapter IV.

I. TRANSFORMATION OF THE QUESTIONNAIRES

Utilizing the procedure outlined in Section G, first the data was recorded and raw frequencies were determined. The relative frequencies and cumulative relative frequencies were then computed and are presented in Appendix C, page 54. With the cumulative relative frequencies established the most difficult part of the procedure was to establish the groups of "like" categories (i.e., instances with responses only in categories A, B, and C are grouped with all other instances with responses only in categories A, B and C). All values of $p_{ij} \geq 0.98$ and $p_{ij} \leq 0.02$ were removed (i.e., all of the 1's and 0's are removed in this study). These values are given in Appendix C.

The next step was to normalize the values and transform them to one scale. This was accomplished using A Programming Language (APL) function that converts the cumulative relative frequencies to a scale value (steps 3 thru 10 as previously discussed).

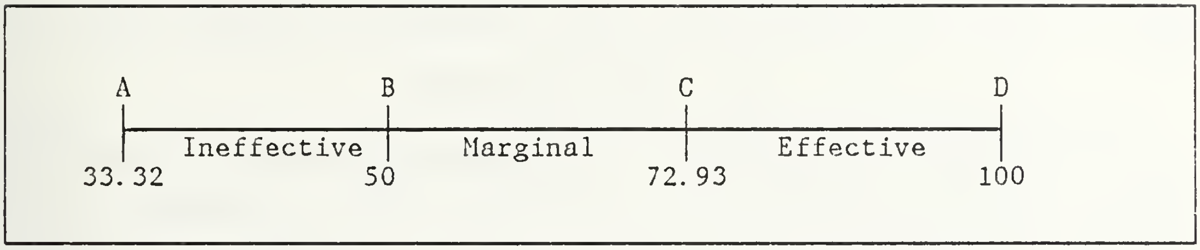


Figure 3. Transformed Category Bounds on Interval Scale

This APL function (called NORM) is given in Appendix I and the scale values of instances are shown in Appendix C. The final step was to transform all of the instances to the same scale. Another APL function (called TRANS) accomplished this transformation of each individual group. To use this transformation two upper category bounds had to be established, the values of these bounds were arbitrarily given as 100 for the upper category bound for category D and 50 as the upper category bound for category B. The group with the categories B, C and D, was transformed first using the transformation program producing a value of the upper category bound for C. These values were then used to solve for the values of the category upper bounds of the other groups. This puts all of the instances on one interval scale with the category upper bounds on the same scale. The transformed values are presented in Appendix C, page 63, and the corresponding category upper bounds are:

- A (Totally Ineffective) = 33.32 ,
- B (Ineffective) = 50.00 ,
- C (Marginal) = 72.933 ,
- D (Effective) = 100.00 .

As seen in Fig. 3 the category upper bounds are now plotted on their interval scale. The transformed values of unit effectiveness were then used to conduct the analysis and determine the function that best fit the data. The analysis is discussed in Chapter IV.

IV. THE ANALYSIS

The preceding chapter explained the techniques used to determine the values of the dependent variable, unit effectiveness. This chapter develops the formulation of the model and shows how the model is used to support the Generalized Value System (GVS). In addition, an analysis of how well the model fits the actual data is explored.

A. REGRESSION TECHNIQUE

Once the transformed values were compiled, an analysis of the data and a determination of the best fitting function of the state variables and the values of unit effectiveness was done with the use of a regression technique in an A Programming Language (APL) package. Several functions that were tested using this regression technique:

- Exponential,
- Logarithmic, and
- Square root.

None of these functions were determined to fit the values of unit effectiveness very well since the standard deviation was unusually large. Appendix E shows the results of the functions tested and the coefficient of determination (R Square) for each. An example of the logarithmic function is presented in Fig. 4.

The next step in finding the best fitting function was to determine the shape of the four dimensional vector of personnel, ammunition, vehicles, and POL. This was done with the use of GRAFSTAT on the main frame computer at the Naval Postgraduate School. Each variable was compared to all other variables by plotting two variables against each other while holding the other two variables constant, using contour surface plots. These plots are presented in Appendix D. From these plots the general shape of the surfaces was determined to be elliptical; thus, the model form was given by:

$$Y = a^2 + b^2(PER - X)^2 + c^2(AMMO - X)^2 + d^2(VEH - X)^2 + e^2(POL - X)^2 \,. \qquad (4.1)$$

Once the general shape of the surface was determined the next step was to perform regression analysis on different forms of these functions to determine the best fitting model. The best fitting model as determined by the regression software (Appendix F) and utilizing the function in Eqn. 4.1 is:

$$Y = 88.978 - .0056 \times X1 - .0055 \times X2 - .0054 \times X3 - .0005 \times X4 \ , \tag{4.2}$$

where:

Y = The values of unit effectiveness,

X1 = $(PER - 100)^2$,

X2 = $(AMMO - 100)^2$,

X3 = $(VEH - 100)^2$, and

X4 = $(POL - 100)^2$.

YY REGRESS ln(PER), AMMO, ln(VEH), POL

ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	3.0142E4	7.5354E3	1.6951E2
RESIDUAL	139	6.1791E3	4.4454E1	
TOTAL	143	3.6321E4		

R SQUARE: 0.786
STD ERROR: 13.6

COEFFICIENTS	T STATISTICS
-537.5068	-14.6404
35.4347	8.4483
43.5474	19.5124
155.2033	8.9433
8.9465	1.0335

VARIANCE-COVARIANCE MATRIX:

1.3479E+3	-9.0701E+1	-2.8662E+1	-4.6951E+2	-1.1899E+2
-9.0701E+1	1.7592E+1	9.8947E-1	5.4825E00	2.5506E00
-2.8662E+1	9.8947E-1	4.9809E00	2.2235E00	1.0626E00
-4.6951E+2	5.4825E00	2.2235E00	3.0116E+2	4.2289E-1
-1.1899E+2	2.5506E00	1.0626E00	4.2289E-1	7.4939E+1

DURBIN-WATSON: 1.45

Figure 4. Regression of the Logarithmic Function

Equation 4.2 has a coefficient of determination (R square) of 84% and a standard deviation of 6.41. It was determined to be the best model among the techniques tested that could be developed from the data base. Dividing Eqn. 4.2 by 88.978 would set the unit effectiveness on a scale from 0 to 1. For the purposes of this thesis this function (Eqn. 4.3) is referred to as the Degraded Power Function (DPF), which is the degradation of power based on the STATE of the entity at time, t:

$$DPF = \frac{88.978 - .0056 \times X1 - .0055 \times X2 - .0054 \times X3 - .0005 \times X4}{88.978} \quad . \tag{4.3}$$

The DPF is now an estimate of the overall unit effectiveness based on the four independent variables, personnel, ammunition, vehicles and POL.

B. WILCOXON SIGN RANK TEST OF LOCATION FOR PAIRED SAMPLES

Based on Eqn. 4.2 a Wilcoxon Signed Rank test was used to determine how well the model fit the actual transformed values. For this test X represents the transformed values and Y represents the predicted values using Eqn. 4.2 . Since the direction of the difference between X and Y could not be anticipated, the hypothesis is set with a two sided alternative for the median of D = X - Y. and it is assumed that the differences, D, are symmetrically distributed about the median of D (M_D). Therefore, the null-hypothesis (H), and alternative hypothesis (A), are:

H: $M_D = 0$, and

A: $M_D \neq 0$.

Using the Wilcoxon Sign Rank test, a table was established to determine the differences between the Transformed values (X), and the predicted data (Y). The sign of the difference between X and Y is used to determine the sign (positive or negative) of the rank of the D_i's = $X_i - Y_i$ for all instances i . The ten steps in this process are explained below.

1. Find the differences between the actual data and the predicted data.
2. Find the absolute value of these differences, noted as $| D_i |$.
3. Rank the $| D_i |$ with the smallest absolute difference having a value of 1, the next smallest a value of 2, etc.
4. Give the sign of the D_i to these ranks.
5. Find the value of $T_+ = \sum$ positive ranks.
6. Find the value of $T_- = \sum$ negative ranks.
7. For $n \leq 15$ determine the larger value of T_+ and T_-
8. For $n > 15$ determine the larger value of $Z_{+,R}$ and $Z_{-,R}$ using Eqn. 4.3 below, and use this value for a right-tail probability look up for the p-value, sometimes called the critical level.
9. Using Table G or A (when using normal approximation), [Ref. 8: pp.123-141] determine the p-value in terms of a right-tail probability for one of (T_+ , T_-), or ($Z_{+,R}$, $Z_{-,R}$ when $n > 15$).

10. This p-value is the basis of acceptance or rejection of the Null Hypothesis (H), which stated the median differences between the transformed values (X) and predicted (Y) values are zero.

Based on the Wilcoxon Signed Rank test, presented in Appendix G with n = 144 the sum of ranks were: $T_- = 5397$ and $T_+ = 5043$, and $Z_{+,R}$ and $Z_{-,R}$ were computed using Eqn. 4.3

$$Z_{+,R} = \frac{T_+ - 0.5 - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}, \qquad Z_{-,R} = \frac{T_- - 0.5 - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}, \tag{4.3}$$

which resulted in $Z_{+,R} = -0.354$ and $Z_{-,R} = 0.352$, $Z_{-,R}$ was used since it had the larger value. Based on the information in a normal CDF table, and since this was a two-sided hypothesis test, a p-value of $2\times(0.3624) = 0.7248$ was computed. Based on this p-value, the null hypothesis could not be rejected and the assumption that the predicted values approximate the actual values was valid. Note that the most deviation was at the large values of X (Appendix G). Since this model will not be used to predict values of the state variables above 100%, this model is considered quite accurate.

C. DEVELOPMENT OF A MAPPING FUNCTION

Recall from Chapter II that Predicted Adjusted Basic Inherent Power (PABIP) may be calculated using Eqn. 4.4

$$PABIP(X1(t)|\vec{SX1}(t_p)) = ABIP(\vec{SX1}(t_p)) \times e^{-L(t-t_p)} \qquad for \quad t_p \le t \le t_E . \tag{4.4}$$

In order to demonstrate how Eqn. 4.4 relates to time, a short example is used to describe how PABIP functions. If the unit has the following constants, the shape that Equation 4.4 takes on is depicted in Fig. 5,

- L = 0.02, a rate of change which may have several components such as attrition, reinforcements, and logistics effects,
- $t_E = 60$, the end of the planning horizon,
- $t_A = 0$, the point in time that a unit is in position to accomplish the mission,
- $t_p = 0$, the current time, and
- ABIP = 1000 .

Note that PABIP(t) = ABIP, when t=0 and the formula for PABIP (Eqn. 4.4) is only based on the value of ABIP at one specific time ($t = t_p$). For $t > t_p$, Eqn. 4.4 uses

an exponential decay to forcast PABIP. Therefore, the state of the unit is only used at time, t_p. The proposed model to replace Eqn. 4.4 is:

$$PABIP(X1(t)|\vec{SX1}(t)) = ABIP(\vec{SX1}(t)) \times DPF(\vec{SX1}(t)), \tag{4.5}$$

thereby eliminating the exponential decay function, $e^{-L(t-t_p)}$, and utilizing the state of the entity at discrete increments of time, t, for $t_p \leq t \leq t_E$.

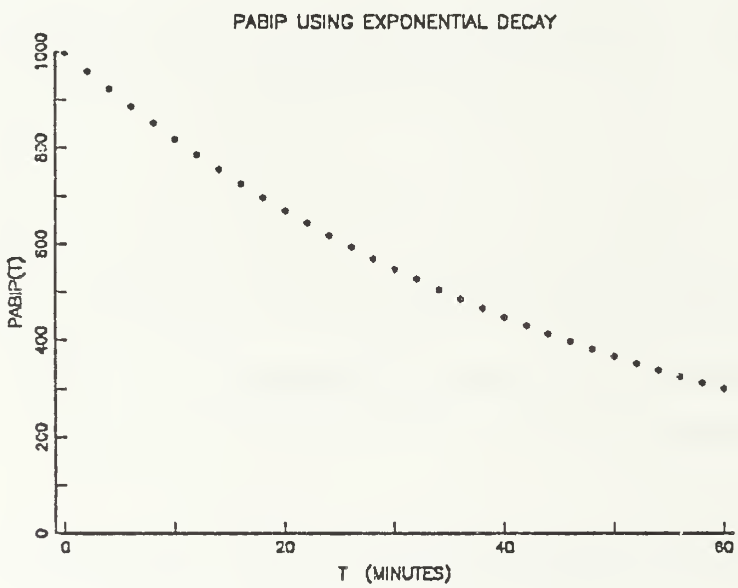


Figure 5. Predicted Basic Inherent Power using Eqn. 4.4

Also from Chapter II the Situational Inherent Power (SIP) is computed by using Equation 4.6

$$SIP(X1(t)|(t_p)) = PABIP(\vec{SX1}(t)|\vec{SX1}(t_p)) \times e^{-D(t_A - t)} \qquad for\ 0 \leq t \leq t_A, \tag{4.6}$$

$$SIP(X1(t)) = PABIP(\vec{SX1}(t)) \qquad for\ t \geq t_A.$$

Recall that the rate at which a unit attains readiness (mission position discount factor), D, accounts for the time until the unit is in position to perform its mission. The proposed model could make the SIP more accurate since it continually uses the forecasted state of the unit to calculate the PABIP. Essentially the PABIP will equal ABIP if the forecasted state of the unit is continually updated, as ALARM has the capability of doing. This can be seen in the following example.

A Blue forces is defending against an attacking Red force. The Blue force has a Basic Inherent Power (BIP) of 1000 and the Red force has a BIP of 2000. Both forces are at 100% strength with the Red force 16,200 meters from the Blue force at $t=0$. Rate of march for Red force is 270 meters per minute, therefore, the Red force is presently 60 minutes from the Blue force's position. In order to use the SIP, D (mission position discount factor) must be computed based on the time it takes the Red forces to reach the position to conduct an attack against the Blue force (for this example this time will be 30 minutes). A negligable amount of power is assigned to the Red force at $t=0$ (i.e., 5% of the Red force PABIP). This grows until the Red force is at 100% of PABIP, when they are 30 minutes from the Blue force's position. Thus the discount factor for the Blue force considering the Red force is determined by solving $0.05 = e^{-D(30)}$ for D, resulting in $D = 0.0998$ / minute. Since the Blue force is in position, their SIP(t) = PABIP(t) for all t. During the conduct of this battle the Blue and Red force's SIP's will be plotted, as well as the difference between their respective SIP's. For the first 30 minutes of the battle the attrition was due mainly to artillery and vehicle break-downs. The rates of attrition for Blue and Red forces are presented in Table 11. Based on the rates shown in Table 11, the values for Blue and Red force's DPF's, PABIP's, SIP's, and the difference between the Blue and Red forces SIP's were computed. These values are presented in Appendix H, and a graphical representation of the battle is shown in Fig. 6.

Table 11. BLUE AND RED FORCES ATTRITION RATES PER MINUTE

Time (Min)	Blue Forces				Red Forces			
	PER	AMMO	VEH	POL	PER	AMMO	VEH	POL
0 - 30	0.333	0.500	0.000	0.333	0.167	0.067	0.067	0.333
30 - 60	1.333	1.500	1.333	0.333	1.700	2.433	1.800	0.333

Note that as the Red force moves into attack position their SIP increases but their SIP is also being degraded by the attrition factors.

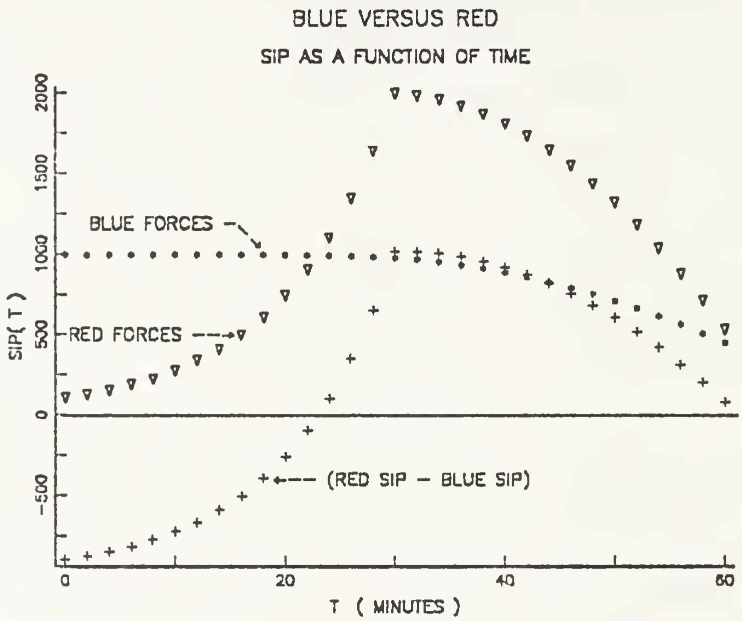


Figure 6. Blue versus Red Battle

Once the Red force begins the attack, the attrition rate increases and their SIP reduces at a greater rate than the Blue force. However, at t = 60 minutes the Red force's SIP is greater than the Blue force's SIP, which would indicate that the Red force would win the battle.

If the Blue commander could forecast the state of both Red and Blue forces at t = 60, he could request that reinforcements arrive prior to the Red attack. The same example is run again with the same attrition rates, presented in Table 11, and the same BIP's for both forces, only this time the Blue force requests that reinforcements arrive by t = 30 minutes. The reinforcements in this example have a BIP of 1000. This second battle is presented in Fig. 7. The Red force's SIP is increases until they are in position to attack and the Blue forces SIP is increasing as the reinforcements move into position

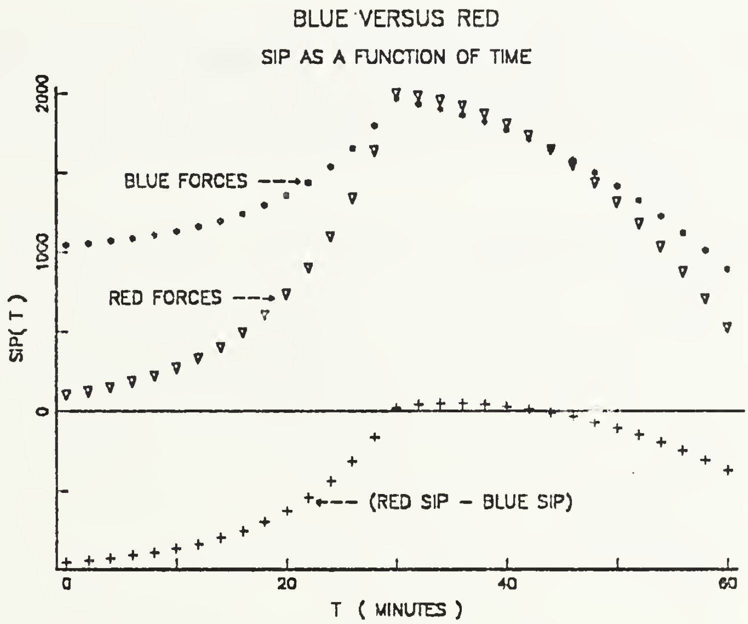


Figure 7. **Blue Versus Red Battle with Reinforcements**

to support the Blue force. At t=30 minutes, both force's SIP's are almost the same. As the Red force attacks, their attrition rate is higher than the Blue force, enabling the Blue force to win this battle. Therefore, if a decision-maker can forecast the anticipated state vector values of the Red and Blue forces over $t_p \leq t \leq t_E$, he can determine by what time resupply or support has to occur in order to positively affect the outcome of a battle.

This example illustrates some very important aspects of representing and modeling the decision process. First, future state forecasting is the only way the Blue force could decide to initiate commitment of the reserve at t=0 to realize success at t=60. That is, decision tables based on state vector values at t=0 could not cause this decision to occur. Secondly, the forecast is based on anticipated force states for $t_p \leq t \leq t_E$ using the DPF and not on an arbitrary power rate of change. Finally, this process is very close

to the process actually used by the S-2 (INTEL), S-3 (OPNS) staff functions in an actual combat unit. These techniques may be usable not only as an aid in systemic combat simulations but as a decision aid to a unit staff.

D. FUTURE APPLICATIONS

The feasibility of developing a mapping function between a multiattribute state vector and a unit's power degradation has been demonstrated using a Pilot experiment. This methodology has potential use both in combat models and as a decision aid to combat unit staff planners.

It is recommended that this same experiment be repeated using subjects currently in mechanized battalions to determine the validity between that group and the Pilot group. Next, additional experiments must be designed and implemented for larger units, different missions, different threat forces, and possibly a revised set of independent variables. For example, considerations of a division will likely require additional independent variables such as Class IX (repair parts). Environmental factors such as weather may also be included.

Obviously the goal is to accomplish the required mapping with a minimum number of independent variables. Hopefully, this thesis provides the basic methodology for conduct and analysis of a large family of experiments required to realize robust mapping functions.

APPENDIX A. SAMPLE QUESTIONAIRE

A. PURPOSE & MOTIVATION

The purpose of this questionaire is to obtain an estimate of the degradation in a units effectiveness based on the threat posed, by the degradation in the 4 key variables of a unit. This questionaire will not just be used as a subject for a thesis and later diregarded. It is to be used to help the S-3 and decision maker to determine if he can handle a mission based on 4 key factors. Your answers are therefore very important to insure a good decision is made. You answers, as the decision maker, of the percent degradation of your units effectiveness based on the changes to these key variables is a measure of the relative importance of each variable to the accomplishment of your mission. Your answers will also help to develop a more accurate and realistic representation of how changes to the key variables effect your view of its relative importance.

1. Key Variables

The 4 key variables used throughout this questionaire are:

% Personnel,

% Ammo,

% Weapon Systems / Combat Vehicles, and

% POL (Fuel) .

2. Instructions

In the remainder of this questionaire, you will be asked to place yourself in the role of the decision maker of the unit and determine how changes to your units fuel, ammo, personnel and vehicles will effect your interpretation of your units ability to accomplish its assigned mission.

Please respond to the questions asked in accordance with your feelings regarding the situation. There is no right or wrong answer to any of the questions. As a decision maker in a combat situation you will be required to make rapid estimates of the situation. Therefore, with this in mind you should only take enough time to fully understand the situation presented and record your response. Once you have recorded a response you should not change your response.

You will receive an example question and two practice questions. You are then asked to answer 48 questions by putting an X under the response you feel best describes the unit's ability to accomplish its assigned mission.

Based on the following situation you will be asked to answer questions on the degree to which you determine your unit is able to accomplish its assigned mission. Use the following situation to answer all of the questions in this survey.

Situation

1. Enemy - 127th Motorized Rifle Regiment.

2. Friendly - Your unit 2nd Bn 41st Inf Mech is currently conducting deliberate defensive operations along the forward line of troops (FLOT).
   - Your unit is presently in prepared defensive positions.
   - Your unit is the forward unit. i.e. no units to your front.
   - Your unit is currently engaged in combat with the enemy.

Mission

Your unit 2/41st Inf (M) will conduct a deliberate defense of present positions for a minimum of 24 hours, longer if possible, to prevent the enemy from controlling this key terrain.

Based on the above scenario and mission answer the following questions.

B. EXAMPLE OF THE RESPONSE FORM

****

THE CURRENT STATUS OF YOUR UNIT IS :

75% PERSONNEL,

25% AMMUNITION,

50% WEAPON SYSTEMS, and

50% POL (FUEL) .

BASED ON THIS STATUS, INDICATE BELOW THE CURRENT EFFECTIVENESS OF YOUR UNIT'S ABILITY TO CONTINUE TO ACCOMPLISH ITS CURRENT MISSION OF DELIBERATE DEFENSE.

TOTALLY EFFECTIVE	EFFECTIVE	MARGINAL	INEFFECTIVE	TOTALLY INEFFECTIVE
( )	( )	( )	( )	( )

****

APPENDIX B. PERSONAL HISTORY FORM

Please complete the following form by entering the appropriate responses:

1) Time on active duty (average): (8) Years, (9) Months.

2) Time spent as a staff officer (average): (3) Years, (2) Months.

3) Present Rank (0 - LTC), (7 - MAJ), (53 - CPT).

4) Were the situations presented understandable? (55 - Yes), (5 - No).

5) Would you say the situations were realistic? (49 - Yes), (11 - No).

6) You may have noticed that the Personnel and Weapon systems are only evaluated at 100%, 75%, and 50%. Would you consider the 50% a minimum level to evaluate these two variables or should one or both be evaluated at 25% ?
(check the statement that applies).

(2) Just Personnel should be evaluated at 25%.

(2) Just Weapon systems should be evaluated at 25%.

(10) Both Personnel and Weapon systems should be evaluated at 25%.

(46) I consider 50% for both a minimum level.

APPENDIX C. TRANSFORMING CATEGORICAL RESPONSES TO INTERVAL SCALE

Recall from Chapter III the procedure for Constructing Interval Scales from Categorical Responses is:

1. Arrange the raw frequency data in a table where the rows are scenarios (instances) and the columns are categories. Columns should be in rank order with the least favorable category in the left column and the most favorable in the right column.
2. Compute the relative cumulative frequencies for each row and record these values in a new table. This table is reffered to as the P array and all values of $p_{ij} \geq 0.98$ and $p_{ij} \leq 0.02$ are removed. This creates an $n \times (m-k)$ array, where k is the number of columns removed.
3. Treating these p_{ij} values as leftward areas under a Normal (0,1) curve, look up the values of Z from a table of the normal distribution. Record these as a new table which will be the Z = $\| z_{ij} \|$ array for the computations that follow.
4. For each instance, i, in the Z array, compute the row average, $\bar{z}_i$
5. For each column j in the Z array, compute the column average, b_j . Note that b_j is the value of the upper bound of category j on the scale being developed.
6. Compute the grand average, $\bar{b}$, of all values of the Z array.
7. Compute B = $\sum_{j1}^{m-k}(b_j - \bar{b})^2$, the sum of squared column differences.
8. For each row compute $A_i = \sum\sum_{j=1}^{m-k}(z_{ij} - \bar{z}_i)^2$, the sum of squared individual differences.
9. For each scenario compute $\sqrt{\frac{B}{A_i}}$, an estimate of $\sqrt{\sigma_i^2 + c}$.
10. Finally for each row (scenario) compute $S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$, for all i.

These S_i 's are the scale values of the instances, on the same interval scale as the category bounds, b_j . Now all instances and category bounds are on the desired scale and any linear transformation $Y = \alpha + \beta \times X,\ \ \beta > 0$, may now be performed to adjust the scale as desired, using the same transformation to move both scenarios and category bounds. [Ref. 7: pp. 14-15.]

The remainder of this appendix shows the steps used to transform the categorical responses to an interval scale.

A. GROUP 1 RAW FREQUENCIES

NO.	A	B	C	D	E
1	4	8	8	0	0
2	2	4	12	2	0
3	0	0	5	9	6
4	2	4	12	2	0
5	1	0	2	12	5
6	4	7	8	1	0
7	1	0	10	7	2
8	1	6	11	2	0
9	4	6	8	2	0
10	0	4	10	6	0
11	1	3	12	4	0
12	2	1	4	12	1
13	1	1	13	5	0
14	0	0	4	12	4
15	3	7	10	0	0
16	1	2	12	5	0
17	1	4	13	2	0
18	1	2	8	9	0
19	0	6	7	7	0
20	0	5	11	4	0
21	3	1	11	5	0
22	3	2	13	2	0
23	3	6	11	0	0
24	3	10	7	0	0
25	1	1	9	9	0
26	1	1	12	6	0
27	1	1	9	9	0
28	1	1	9	8	1
29	0	4	9	7	0
30	2	2	10	6	0
31	0	0	5	15	0
32	3	1	7	9	0
33	1	4	12	3	0
34	1	5	11	3	0
35	0	0	2	10	8
36	4	9	7	0	0
37	3	10	6	1	0
38	0	0	4	15	1
39	3	1	9	7	0
40	1	3	14	2	0
41	0	2	9	9	0
42	3	3	12	2	0
43	1	1	8	10	0
44	4	10	6	0	0
45	4	10	6	0	0
46	0	0	5	13	2
47	1	2	14	3	0
48	0	2	1	16	1

B. GROUP 2 RAW FREQUENCIES

NO.	A	B	C	D	E
49	0	3	12	5	0
50	1	4	14	1	0
51	1	2	13	4	0
52	0	1	3	11	5
53	0	0	1	15	4
54	0	6	12	2	0
55	0	0	12	8	0
56	1	3	16	0	0
57	1	2	17	0	0
58	0	1	4	14	1
59	0	0	1	13	6
60	0	2	13	5	0
61	1	7	11	1	0
62	0	3	15	2	0
63	0	0	4	6	10
64	0	2	10	8	0
65	2	14	3	1	0
66	0	1	14	4	1
67	0	0	0	9	11
68	0	7	12	1	0
69	0	0	3	13	4
70	0	10	10	0	0
71	0	1	13	5	1
72	0	2	16	2	0
73	2	12	4	2	0
74	3	14	3	0	0
75	2	11	6	1	0
76	2	12	3	3	0
77	0	3	11	6	0
78	0	6	13	1	0
79	4	13	3	0	0
80	0	3	14	3	0
81	0	9	10	1	0
82	0	6	10	4	0
83	0	4	9	7	0
84	0	0	4	15	1
85	0	4	10	6	0
86	2	13	5	0	0
87	5	11	4	0	0
88	0	0	0	15	5
89	0	3	13	4	0
90	1	13	3	3	0
91	5	13	2	0	0
92	0	1	13	5	1
93	0	0	0	12	8
94	0	0	2	14	4
95	1	12	3	4	0
96	1	11	5	2	1

C. GROUP 3 RAW FREQUENCIES

NO.	A	B	C	D	E
97	1	12	4	3	0
98	3	14	3	0	0
99	1	16	3	0	0
100	0	3	7	10	0
101	1	16	3	0	0
102	2	15	3	0	0
103	1	12	7	0	0
104	0	1	11	8	0
105	0	2	10	8	0
106	0	0	0	13	7
107	2	14	3	1	0
108	0	1	5	12	2
109	0	1	13	5	1
110	0	0	12	6	2
111	0	1	13	6	0
112	0	1	6	12	1
113	0	2	11	7	0
114	0	0	10	10	0
115	1	16	3	0	0
116	0	1	11	8	0
117	0	0	0	17	3
118	0	0	0	0	20
119	0	2	12	6	0
120	1	15	3	1	0
121	0	1	18	1	0
122	0	2	14	4	0
123	0	0	18	2	0
124	0	1	14	5	0
125	0	0	0	9	11
126	1	12	7	0	0
127	0	1	4	12	3
128	1	16	3	0	0
129	0	0	4	14	2
130	0	5	15	0	0
131	1	14	5	0	0
132	0	1	15	4	0
133	0	0	4	15	1
134	0	1	15	4	0
135	1	14	5	0	0
136	0	6	14	0	0
137	0	5	15	0	0
138	1	16	3	0	0
139	1	16	3	0	0
140	0	2	17	1	0
141	0	2	18	0	0
142	0	0	0	3	17
143	1	17	2	0	0
144	0	0	0	15	5

D. GROUP 1 RELATIVE FREQUENCIES

NUMBER	A	B	C	D	E
1	0.2	0.4	0.4	0	0
2	0.1	0.2	0.6	0.1	0
3	0	0	0.25	0.45	0.3
4	0.1	0.2	0.6	0.1	0
5	0.05	0	0.1	0.6	0.25
6	0.2	0.35	0.4	0.05	0
7	0.05	0	0.5	0.35	0.1
8	0.05	0.3	0.55	0.1	0
9	0.2	0.3	0.4	0.1	0
10	0	0.2	0.5	0.3	0
11	0.05	0.15	0.6	0.2	0
12	0.1	0.05	0.2	0.6	0.05
13	0.05	0.05	0.65	0.25	0
14	0	0	0.2	0.6	0.2
15	0.15	0.35	0.5	0	0
16	0.05	0.1	0.6	0.25	0
17	0.05	0.2	0.65	0.1	0
18	0.05	0.1	0.4	0.45	0
19	0	0.3	0.35	0.35	0
20	0	0.25	0.55	0.2	0
21	0.15	0.05	0.55	0.25	0
22	0.15	0.1	0.65	0.1	0
23	0.15	0.3	0.55	0	0
24	0.15	0.5	0.35	0	0
25	0.05	0.05	0.45	0.45	0
26	0.05	0.05	0.6	0.3	0
27	0.05	0.05	0.45	0.45	0
28	0.05	0.05	0.45	0.4	0.05
29	0	0.2	0.45	0.35	0
30	0.1	0.1	0.5	0.3	0
31	0	0	0.25	0.75	0
32	0.15	0.05	0.35	0.45	0
33	0.05	0.2	0.6	0.15	0
34	0.05	0.25	0.55	0.15	0
35	0	0	0.1	0.5	0.4
36	0.2	0.45	0.35	0	0
37	0.15	0.5	0.3	0.05	0
38	0	0	0.2	0.75	0.05
39	0.15	0.05	0.45	0.35	0
40	0.05	0.15	0.7	0.1	0
41	0	0.1	0.45	0.45	0
42	0.15	0.15	0.6	0.1	0
43	0.05	0.05	0.4	0.5	0
44	0.2	0.5	0.3	0	0
45	0.2	0.5	0.3	0	0
46	0	0	0.25	0.65	0.1
47	0.05	0.1	0.7	0.15	0
48	0	0.1	0.05	0.8	0.05

E. GROUP 2 RELATIVE FREQUENCIES

NUMBER	A	B	C	D	E
49	0	0.15	0.6	0.25	0
50	0.05	0.2	0.7	0.05	0
51	0.05	0.1	0.65	0.2	0
52	0	0.05	0.15	0.55	0.25
53	0	0	0.05	0.75	0.2
54	0	0.3	0.6	0.1	0
55	0	0	0.6	0.4	0
56	0.05	0.15	0.8	0	0
57	0.05	0.1	0.85	0	0
58	0	0.05	0.2	0.7	0.05
59	0	0	0.05	0.65	0.3
60	0	0.1	0.65	0.25	0
61	0.05	0.35	0.55	0.05	0
62	0	0.15	0.75	0.1	0
63	0	0	0.2	0.3	0.5
64	0	0.1	0.5	0.4	0
65	0.1	0.7	0.15	0.05	0
66	0	0.05	0.7	0.2	0.05
67	0	0	0	0.45	0.55
68	0	0.35	0.6	0.05	0
69	0	0	0.15	0.65	0.2
70	0	0.5	0.5	0	0
71	0	0.05	0.65	0.25	0.05
72	0	0.1	0.8	0.1	0
73	0.1	0.6	0.2	0.1	0
74	0.15	0.7	0.15	0	0
75	0.1	0.55	0.3	0.05	0
76	0.1	0.6	0.15	0.15	0
77	0	0.15	0.55	0.3	0
78	0	0.3	0.65	0.05	0
79	0.2	0.65	0.15	0	0
80	0	0.15	0.7	0.15	0
81	0	0.45	0.5	0.05	0
82	0	0.3	0.5	0.2	0
83	0	0.2	0.45	0.35	0
84	0	0	0.2	0.75	0.05
85	0	0.2	0.5	0.3	0
86	0.1	0.65	0.25	0	0
87	0.25	0.55	0.2	0	0
88	0	0	0	0.75	0.25
89	0	0.15	0.65	0.2	0
90	0.05	0.65	0.15	0.15	0
91	0.25	0.65	0.1	0	0
92	0	0.05	0.65	0.25	0.05
93	0	0	0	0.6	0.4
94	0	0	0.1	0.7	0.2
95	0.05	0.6	0.15	0.2	0
96	0.05	0.55	0.25	0.1	0.05

F. GROUP 3 RELATIVE FREQUENCIES

NUMBER	A	B	C	D	E
97	0.05	0.6	0.20	0.15	0
98	0.15	0.7	0.15	0	0
99	0.05	0.8	0.15	0	0
100	0	0.15	0.35	0.5	0
101	0.05	0.8	0.15	0	0
102	0.1	0.75	0.15	0	0
103	0.05	0.6	0.35	0	0
104	0	0.05	0.55	0.4	0
105	0	0.1	0.5	0.4	0
106	0	0	0	0.65	0.35
107	0.1	0.7	0.15	0.05	0
108	0	0.05	0.25	0.6	0.1
109	0	0.05	0.65	0.25	0.05
110	0	0	0.6	0.3	0.1
111	0	0.05	0.65	0.3	0
112	0	0.05	0.3	0.6	0.05
113	0	0.1	0.55	0.35	0
114	0	0	0.5	0.5	0
115	0.05	0.8	0.15	0	0
116	0	0.05	0.55	0.4	0
117	0	0	0	0.85	0.15
118	0	0	0	0	1
119	0	0.1	0.6	0.3	0
120	0.05	0.75	0.15	0.05	0
121	0	0.05	0.9	0.05	0
122	0	0.1	0.7	0.2	0
123	0	0	0.9	0.1	0
124	0	0.05	0.7	0.25	0
125	0	0	0	0.45	0.55
126	0.05	0.6	0.35	0	0
127	0	0.05	0.2	0.6	0.15
128	0.05	0.8	0.15	0	0
129	0	0	0.2	0.7	0.1
130	0	0.25	0.75	0	0
131	0.05	0.7	0.25	0	0
132	0	0.05	0.75	0.2	0
133	0	0	0.2	0.75	0.05
134	0	0.05	0.75	0.2	0
135	0.05	0.7	0.25	0	0
136	0	0.3	0.7	0	0
137	0	0.25	0.75	0	0
138	0.05	0.8	0.15	0	0
139	0.05	0.8	0.15	0	0
140	0	0.1	0.85	0.05	0
141	0	0.1	0.9	0	0
142	0	0	0	0.15	0.85
143	0.05	0.85	0.1	0	0
144	0	0	0	0.75	0.25

G. GROUP 1 CUMULATIVE RELATIVE FREQUENCIES

NUMBER	A	B	C	D	E
1	0.2	0.6	1	1	1
2	0.1	0.3	0.9	1	1
3	0	0	0.25	0.7	1
4	0.1	0.3	0.9	1	1
5	0.05	0.05	0.15	0.75	1
6	0.2	0.55	0.95	1	1
7	0.05	0.05	0.55	0.9	1
8	0.05	0.35	0.9	1	1
9	0.2	0.5	0.9	1	1
10	0	0.2	0.7	1	1
11	0.05	0.2	0.8	1	1
12	0.1	0.15	0.35	0.95	1
13	0.05	0.1	0.75	1	1
14	0	0	0.2	0.8	1
15	0.15	0.5	1	1	1
16	0.05	0.15	0.75	1	1
17	0.05	0.25	0.9	1	1
18	0.05	0.15	0.55	1	1
19	0	0.3	0.65	1	1
20	0	0.25	0.8	1	1
21	0.15	0.2	0.75	1	1
22	0.15	0.25	0.9	1	1
23	0.15	0.45	1	1	1
24	0.15	0.65	1	1	1
25	0.05	0.1	0.55	1	1
26	0.05	0.1	0.7	1	1
27	0.05	0.1	0.55	1	1
28	0.05	0.1	0.55	0.95	1
29	0	0.2	0.65	1	1
30	0.1	0.2	0.7	1	1
31	0	0	0.25	1	1
32	0.15	0.2	0.55	1	1
33	0.05	0.25	0.85	1	1
34	0.05	0.3	0.85	1	1
35	0	0	0.1	0.6	1
36	0.2	0.65	1	1	1
37	0.15	0.65	0.95	1	1
38	0	0	0.2	0.95	1
39	0.15	0.2	0.65	1	1
40	0.05	0.2	0.9	1	1
41	0	0.1	0.55	1	1
42	0.15	0.3	0.9	1	1
43	0.05	0.1	0.5	1	1
44	0.2	0.7	1	1	1
45	0.2	0.7	1	1	1
46	0	0	0.25	0.9	1
47	0.05	0.15	0.85	1	1
48	0	0.1	0.15	0.95	1

H. GROUP 2 CUMULATIVE RELATIVE FREQUENCIES

NUMBER	A	B	C	D	E
49	0	0.15	0.75	1	1
50	0.05	0.25	0.95	1	1
51	0.05	0.15	0.8	1	1
52	0	0.05	0.2	0.75	1
53	0	0	0.05	0.8	1
54	0	0.3	0.9	1	1
55	0	0	0.6	1	1
56	0.05	0.2	1	1	1
57	0.05	0.15	1	1	1
58	0	0.05	0.25	0.95	1
59	0	0	0.05	0.7	1
60	0	0.1	0.75	1	1
61	0.05	0.4	0.95	1	1
62	0	0.15	0.9	1	1
63	0	0	0.2	0.5	1
64	0	0.1	0.6	1	1
65	0.1	0.8	0.95	1	1
66	0	0.05	0.75	0.95	1
67	0	0	0	0.45	1
68	0	0.35	0.95	1	1
69	0	0	0.15	0.8	1
70	0	0.5	1	1	1
71	0	0.05	0.7	0.95	1
72	0	0.1	0.9	1	1
73	0.1	0.7	0.9	1	1
74	0.15	0.85	1	1	1
75	0.1	0.65	0.95	1	1
76	0.1	0.7	0.85	1	1
77	0	0.15	0.7	1	1
78	0	0.3	0.95	1	1
79	0.2	0.85	1	1	1
80	0	0.15	0.85	1	1
81	0	0.45	0.95	1	1
82	0	0.3	0.8	1	1
83	0	0.2	0.65	1	1
84	0	0	0.2	0.95	1
85	0	0.2	0.7	1	1
86	0.1	0.75	1	1	1
87	0.25	0.8	1	1	1
88	0	0	0	0.75	1
89	0	0.15	0.8	1	1
90	0.05	0.7	0.85	1	1
91	0.25	0.9	1	1	1
92	0	0.05	0.7	0.95	1
93	0	0	0	0.6	1
94	0	0	0.1	0.8	1
95	0.05	0.65	0.8	1	1
96	0.05	0.6	0.85	0.95	1

I. GROUP 3 CUMULATIVE RELATIVE FREQUENCIES

NUMBER	A	B	C	D	E
97	0.05	0.65	0.85	1	1
98	0.15	0.85	1	1	1
99	0.05	0.85	1	1	1
100	0	0.15	0.5	1	1
101	0.05	0.85	1	1	1
102	0.1	0.85	1	1	1
103	0.05	0.65	1	1	1
104	0	0.05	0.6	1	1
105	0	0.1	0.6	1	1
106	0	0	0	0.65	1
107	0.1	0.8	0.95	1	1
108	0	0.05	0.3	0.9	1
109	0	0.05	0.7	0.95	1
110	0	0	0.6	0.9	1
111	0	0.05	0.7	1	1
112	0	0.05	0.35	0.95	1
113	0	0.1	0.65	1	1
114	0	0	0.5	1	1
115	0.05	0.85	1	1	1
116	0	0.05	0.6	1	1
117	0	0	0	0.85	1
118	0	0	0	0	1
119	0	0.1	0.7	1	1
120	0.05	0.8	0.95	1	1
121	0	0.05	0.95	1	1
122	0	0.1	0.8	1	1
123	0	0	0.9	1	1
124	0	0.05	0.75	1	1
125	0	0	0	0.45	1
126	0.05	0.65	1	1	1
127	0	0.05	0.25	0.85	1
128	0.05	0.85	1	1	1
129	0	0	0.2	0.9	1
130	0	0.25	1	1	1
131	0.05	0.75	1	1	1
132	0	0.05	0.8	1	1
133	0	0	0.2	0.95	1
134	0	0.05	0.8	1	1
135	0.05	0.75	1	1	1
136	0	0.3	1	1	1
137	0	0.25	1	1	1
138	0.05	0.85	1	1	1
139	0.05	0.85	1	1	1
140	0	0.1	0.95	1	1
141	0	0.1	1	1	1
142	0	0	0	0.15	1
143	0.05	0.9	1	1	1
144	0	0	0	0.75	1

J. GROUP 1: COMBINE CATEGORIES AND REMOVE 0'S AND 1'S

GRP11

NUMBER	A	B	C
2	0.1	0.3	0.9
4	0.1	0.3	0.9
6	0.2	0.55	0.95
8	0.05	0.35	0.9
9	0.2	0.5	0.9
11	0.05	0.2	0.8
13	0.05	0.1	0.75
16	0.05	0.15	0.75
17	0.05	0.25	0.9
18	0.05	0.15	0.55
21	0.15	0.2	0.75
22	0.15	0.25	0.9
25	0.05	0.1	0.55
26	0.05	0.1	0.7
27	0.05	0.1	0.55
30	0.1	0.2	0.7
32	0.15	0.2	0.55
33	0.05	0.25	0.85
34	0.05	0.3	0.85
37	0.15	0.65	0.95
39	0.15	0.2	0.65
40	0.05	0.2	0.9
42	0.15	0.3	0.9
43	0.05	0.1	0.5
47	0.05	0.15	0.85

GRP12

NUMBER	A	B
1	0.2	0.6
15	0.15	0.5
23	0.15	0.45
24	0.15	0.65
36	0.2	0.65
44	0.2	0.7
45	0.2	0.7

GRP13

NUMBER	A	B	C	D
5	0.05	0.05	0.15	0.75
7	0.05	0.05	0.55	0.9
12	0.1	0.15	0.35	0.95
28	0.05	0.1	0.55	0.95

GRP14

NUMBER	B	C
10	0.2	0.7
19	0.3	0.65
20	0.25	0.8
29	0.2	0.65
41	0.1	0.55

GRP15

NUMBER	C	D
3	0.25	0.7
14	0.2	0.8
31	0.25	0.95
35	0.1	0.6
38	0.2	0.95
46	0.25	0.9
48	0.25	0.95

K. GROUP 2: COMBINE CATEGORIES AND REMOVE 0'S AND 1'S

GRP21

NUMBER	A	B	C
50	0.05	0.25	0.95
51	0.05	0.15	0.8
61	0.05	0.4	0.95
65	0.1	0.8	0.95
73	0.1	0.7	0.9
75	0.1	0.65	0.95
76	0.1	0.7	0.85
90	0.05	0.7	0.85
95	0.05	0.65	0.8
96	0.05	0.6	0.95

GRP22

NUMBER	A	B
56	0.05	0.2
57	0.05	0.15
74	0.15	0.85
79	0.2	0.85
86	0.1	0.75
87	0.25	0.8
91	0.25	0.9

GRP23

NUMBER	B	C
49	0.15	0.75
54	0.3	0.9
60	0.1	0.75
62	0.15	0.9
64	0.1	0.6
68	0.35	0.95
72	0.1	0.9
77	0.15	0.7
78	0.3	0.95
80	0.15	0.85
81	0.45	0.95
82	0.3	0.8
83	0.2	0.65
85	0.2	0.7
89	0.15	0.8

GRP24

NUMBER	B	C	D
52	0.05	0.2	0.75
58	0.05	0.25	0.95
66	0.05	0.75	0.95
71	0.05	0.7	0.95
92	0.05	0.7	0.95

	GRP25	
NUMBER	C	D
53	0.05	0.8
55	0.6	0.95
59	0.05	0.7
63	0.2	0.5
69	0.15	0.8
84	0.2	0.95
93	0.05	0.55

	GRP26
NUMBER	D
67	0.45
88	0.75
94	0.9

L. GROUP 3: COMBINE CATEGORIES AND REMOVE 0'S AND 1'S

	GRP31	
NUMBER	A	B
98	0.15	0.85
99	0.05	0.85
101	0.05	0.85
102	0.1	0.85
103	0.05	0.65
115	0.05	0.85
126	0.05	0.65
128	0.05	0.85
131	0.05	0.75
135	0.05	0.75
138	0.05	0.85
139	0.05	0.85
143	0.05	0.9

	GRP32	
NUMBER	B	C
100	0.15	0.5
104	0.05	0.6
105	0.1	0.6
111	0.05	0.7
113	0.1	0.65
116	0.05	0.6
119	0.1	0.7
121	0.05	0.95
122	0.1	0.8
123	0.05	0.95
124	0.05	0.75
132	0.05	0.8
134	0.05	0.8
140	0.1	0.95

	GRP33
NUMBER	D
106	0.65
117	0.85
125	0.45
142	0.15
144	0.75

NUMBER	GRP34 B
130	0.25
136	0.3
137	0.25
141	0.1

NUMBER	GRP35 C	D
110	0.6	0.9
129	0.2	0.9
133	0.2	0.95

NUMBER	B	GRP36 C	D
108	0.05	0.3	0.9
109	0.05	0.7	0.95
112	0.05	0.35	0.95
127	0.05	0.25	0.85

NUMBER	A	GRP38 B	C
97	0.05	0.65	0.85
107	0.1	0.8	0.95
120	0.05	0.8	0.95

M. COMBINE THE THREE GROUPS A , B , AND C CATEGORIES

No.	Group A B C A	B	C
2	0.1	0.3	0.9
4	0.1	0.3	0.9
6	0.2	0.55	0.95
8	0.05	0.35	0.9
9	0.2	0.5	0.9
11	0.05	0.2	0.8
13	0.05	0.1	0.75
16	0.05	0.15	0.75
17	0.05	0.25	0.9
18	0.05	0.15	0.55
21	0.15	0.2	0.75
22	0.15	0.25	0.9
25	0.05	0.1	0.55
26	0.05	0.1	0.7
27	0.05	0.1	0.55
30	0.1	0.2	0.7
32	0.15	0.2	0.55
33	0.05	0.25	0.85
34	0.05	0.3	0.85
37	0.15	0.65	0.95
39	0.15	0.2	0.65
40	0.05	0.2	0.9
42	0.15	0.3	0.9
43	0.05	0.1	0.5
47	0.05	0.15	0.85
50	0.05	0.25	0.95
51	0.05	0.15	0.8
61	0.05	0.4	0.95
65	0.1	0.8	0.95
73	0.1	0.7	0.9
75	0.1	0.65	0.95
76	0.1	0.7	0.85
90	0.05	0.7	0.85
95	0.05	0.65	0.8
96	0.05	0.6	0.95
97	0.05	0.65	0.85
107	0.1	0.8	0.95
120	0.05	0.8	0.95

N. COMBINE THE THREE GROUPS B , C , AND D CATEGORIES

Group B C D

No.	B	C	D
5	0.1	0.15	0.75
7	0.1	0.55	0.9
12	0.25	0.35	0.95
28	0.15	0.55	0.95
52	0.05	0.2	0.75
58	0.05	0.25	0.95
66	0.05	0.75	0.95
71	0.05	0.7	0.95
92	0.05	0.7	0.95
108	0.05	0.3	0.9
109	0.05	0.7	0.95
112	0.05	0.35	0.95
127	0.05	0.25	0.85

O. COMBINE THE THREE GROUPS A AND B CATEGORIES

Group A B

No.	A	B
1	0.2	0.6
15	0.15	0.5
23	0.15	0.45
24	0.15	0.65
36	0.2	0.65
44	0.2	0.7
45	0.2	0.7
56	0.05	0.2
57	0.05	0.15
74	0.15	0.85
79	0.2	0.85
86	0.1	0.75
87	0.25	0.8
91	0.25	0.9
98	0.15	0.85
99	0.05	0.85
101	0.05	0.85
102	0.1	0.85
103	0.05	0.65
115	0.05	0.85
126	0.05	0.65
128	0.05	0.85
131	0.05	0.75
135	0.05	0.75
138	0.05	0.85
139	0.05	0.85
143	0.05	0.9

P. COMBINE THE THREE GROUPS B AND C CATEGORIES

No.	Group B C B	C
10	0.2	0.7
19	0.3	0.65
20	0.25	0.8
29	0.2	0.65
41	0.1	0.55
49	0.15	0.75
54	0.3	0.9
60	0.1	0.75
62	0.15	0.9
64	0.1	0.6
68	0.35	0.95
72	0.1	0.9
77	0.15	0.7
78	0.3	0.95
80	0.15	0.85
81	0.45	0.95
82	0.3	0.8
83	0.2	0.65
85	0.2	0.7
89	0.15	0.8
100	0.15	0.5
104	0.05	0.6
105	0.1	0.6
111	0.05	0.7
113	0.1	0.65
116	0.05	0.6
119	0.1	0.7
121	0.05	0.95
122	0.1	0.8
123	0.05	0.95
124	0.05	0.75
132	0.05	0.8
134	0.05	0.8
140	0.1	0.95

Q. COMBINE THE THREE GROUPS C AND D CATEGORIES

Group C D No.	C	D
3	0.25	0.7
14	0.2	0.8
31	0.25	0.95
35	0.1	0.6
38	0.2	0.95
46	0.25	0.9
48	0.25	0.95
53	0.05	0.8
55	0.6	0.95
59	0.05	0.7
63	0.2	0.5
69	0.15	0.8
84	0.2	0.95
93	0.05	0.55
94	0.1	0.8
110	0.6	0.9
129	0.2	0.9
133	0.2	0.95

R. NORMALIZING GROUP A B C

NORMALIZED VALUES			ROW AVERAGE
-1.282E0	-5.240E-1	1.282E0	-1.747E-1
-1.282E0	-5.240E-1	1.282E0	-1.747E-1
-8.415E-1	1.254E-1	1.645E0	3.097E-1
-1.645E0	-3.849E-1	1.282E0	-2.495E-1
-8.415E-1	1.010E-7	1.282E0	1.468E-1
-1.645E0	-8.415E-1	8.415E-1	-5.484E-1
-1.645E0	-1.282E0	6.742E-1	-7.509E-1
-1.645E0	-1.036E0	6.742E-1	-6.692E-1
-1.645E0	-6.742E-1	1.282E0	-3.459E-1
-1.645E0	-1.036E0	1.254E-1	-8.521E-1
-1.036E0	-8.415E-1	6.742E-1	-4.012E-1
-1.036E0	-6.742E-1	1.282E0	-1.430E-1
-1.645E0	-1.282E0	1.254E-1	-9.339E-1
-1.645E0	-1.282E0	5.240E-1	-8.010E-1
-1.645E0	-1.282E0	1.254E-1	-9.339E-1
-1.282E0	-8.415E-1	5.240E-1	-5.331E-1
-1.036E0	-8.415E-1	1.254E-1	-5.842E-1
-1.645E0	-6.742E-1	1.036E0	-4.277E-1
-1.645E0	-5.240E-1	1.036E0	-3.776E-1
-1.036E0	3.849E-1	1.645E0	3.312E-1
-1.036E0	-8.415E-1	3.849E-1	-4.977E-1
-1.645E0	-8.415E-1	1.282E0	-4.016E-1
-1.036E0	-5.240E-1	1.282E0	-9.290E-2
-1.645E0	-1.282E0	1.010E-7	-9.756E-1
-1.645E0	-1.036E0	1.036E0	-5.484E-1
-1.645E0	-6.742E-1	1.645E0	-2.247E-1
-1.645E0	-1.036E0	8.415E-1	-6.134E-1
-1.645E0	-2.529E-1	1.645E0	-8.431E-2
-1.282E0	8.415E-1	1.645E0	4.016E-1
-1.282E0	5.240E-1	1.282E0	1.747E-1
-1.282E0	3.849E-1	1.645E0	2.495E-1
-1.282E0	5.240E-1	1.036E0	9.290E-2
-1.645E0	5.240E-1	1.036E0	-2.826E-2
-1.645E0	3.849E-1	8.415E-1	-1.396E-1
-1.645E0	2.529E-1	1.645E0	8.431E-2
-1.645E0	3.849E-1	1.036E0	-7.463E-2
-1.282E0	8.415E-1	1.645E0	4.016E-1
-1.645E0	8.415E-1	1.645E0	2.805E-1

COLUMN AVERAGES		
-1.43	-0.3952	1.027

GRAND AVERAGE
-0.266

Scale Values for Group A B C

$$S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$$

Scale Values	Grand Average	Row Average	(B/A(i))*.5
-0.1024	-0.266	-0.1747	0.937
-0.1024	-0.266	-0.1747	0.937
-0.5709	-0.266	0.3097	0.9844
-0.05637	-0.266	-0.2495	0.8405
-0.4354	-0.266	0.1468	1.154
0.2672	-0.266	-0.5484	0.9724
0.4767	-0.266	-0.7509	0.9891
0.4206	-0.266	-0.6692	1.026
0.02026	-0.266	-0.3459	0.8277
0.9028	-0.266	-0.8521	1.372
0.2627	-0.266	-0.4012	1.318
-0.1246	-0.266	-0.143	0.9895
0.9662	-0.266	-0.9339	1.319
0.5847	-0.266	-0.801	1.062
0.9662	-0.266	-0.9339	1.319
0.4326	-0.266	-0.5331	1.311
0.8925	-0.266	-0.5842	1.983
0.1226	-0.266	-0.4277	0.9088
0.07992	-0.266	-0.3776	0.9162
-0.5707	-0.266	0.3312	0.9197
0.531	-0.266	-0.4977	1.601
0.06171	-0.266	-0.4016	0.816
-0.1719	-0.266	-0.0929	1.013
1.127	-0.266	-0.9756	1.428
0.2153	-0.266	-0.5484	0.8777
-0.102	-0.266	-0.2247	0.7299
0.3179	-0.266	-0.6134	0.952
-0.203	-0.266	-0.08431	0.7471
-0.5938	-0.266	0.4016	0.816
-0.4297	-0.266	0.1747	0.937
-0.4757	-0.266	0.2495	0.8405
-0.3602	-0.266	0.0929	1.013
-0.2415	-0.266	-0.02826	0.8668
-0.1359	-0.266	-0.1396	0.9322
-0.329	-0.266	0.08431	0.7471
-0.2002	-0.266	-0.07463	0.8822
-0.5938	-0.266	0.4016	0.816
-0.4678	-0.266	0.2805	0.7193

S. NORMALIZING GROUP B C D

NORMALIZED VALUES			ROW AVERAGE
-1.282	-1.036	0.6742	-0.548
-1.282	0.1254	1.282	0.04179
-0.6742	-0.3849	1.645	0.1954
-1.036	0.1254	1.645	0.2447
-1.645	-0.8415	0.6742	-0.6042
-1.645	-0.6742	1.645	-0.2247
-1.645	0.6742	1.645	0.2247
-1.645	0.524	1.645	0.1747
-1.645	0.524	1.645	0.1747
-1.645	-0.524	1.282	-0.2958
-1.645	0.524	1.645	0.1747
-1.645	-0.3849	1.645	-0.1283
-1.645	-0.6742	1.036	-0.4277

COLUMN AVERAGES		
-1.468	-0.1556	1.393

GRAND AVERAGE
-0.07677

Scale Values of Group B C D

$$S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$$

Scale Values	Grand Average	Row Average	(B/A(i))*.5
0.6597	-0.07677	-0.548	1.344
-0.1234	-0.07677	0.04179	1.116
-0.2981	-0.07677	0.1954	1.133
-0.3374	-0.07677	0.2447	1.065
0.6578	-0.07677	-0.6042	1.216
0.1136	-0.07677	-0.2247	0.8471
-0.2671	-0.07677	0.2247	0.8471
-0.2263	-0.07677	0.1747	0.8561
-0.2263	-0.07677	0.1747	0.8561
0.2101	-0.07677	-0.2958	0.9697
-0.2263	-0.07677	0.1747	0.8561
0.03389	-0.07677	-0.1283	0.8626
0.3743	-0.07677	-0.4277	1.055

T. NORMALIZING GROUP A B

NORMALIZED VALUES		ROW AVERAGE
-8.415E-1	2.529E-1	-2.943E-1
-1.036E0	1.010E-7	-5.182E-1
-1.036E0	-1.254E-1	-5.809E-1
-1.036E0	3.849E-1	-3.258E-1
-8.415E-1	3.849E-1	-2.283E-1
-8.415E-1	5.240E-1	-1.587E-1
-8.415E-1	5.240E-1	-1.587E-1
-1.645E0	-8.415E-1	-1.243E0
-1.645E0	-1.036E0	-1.341E0
-1.036E0	1.036E0	0.000E0
-8.415E-1	1.036E0	9.749E-2
-1.282E0	6.742E-1	-3.038E-1
-6.742E-1	8.415E-1	8.363E-2
-6.742E-1	1.282E0	3.038E-1
-1.036E0	1.036E0	0.000E0
-1.645E0	1.036E0	-3.044E-1
-1.645E0	1.036E0	-3.044E-1
-1.282E0	1.036E0	-1.226E-1
-1.645E0	3.849E-1	-6.302E-1
-1.645E0	1.036E0	-3.044E-1
-1.645E0	3.849E-1	-6.302E-1
-1.645E0	1.036E0	-3.044E-1
-1.645E0	6.742E-1	-4.855E-1
-1.645E0	6.742E-1	-4.855E-1
-1.645E0	1.036E0	-3.044E-1
-1.645E0	1.036E0	-3.044E-1
-1.645E0	1.282E0	-1.817E-1

COLUMN AVERAGES	
-1.285	0.6159

GRAND AVERAGE
-0.3344

Scale Values of Group A B

$$S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$$

Scale Values	Grand Average	Row Average	(B/A(i))*.5
0.1766	-0.3344	-0.2943	1.737
0.6159	-0.3344	-0.5182	1.834
0.8775	-0.3344	-0.5809	2.086
0.1012	-0.3344	-0.3258	1.337
0.01938	-0.3344	-0.2283	1.55
-0.1135	-0.3344	-0.1587	1.392
-0.1135	-0.3344	-0.1587	1.392
2.606	-0.3344	-1.243	2.365
3.852	-0.3344	-1.341	3.122
-0.3344	-0.3344	0	0.9169
-0.4331	-0.3344	0.09749	1.012
-0.03926	-0.3344	-0.3038	0.9718
-0.4393	-0.3344	0.08363	1.254
-0.6296	-0.3344	0.3038	0.9718
-0.3344	-0.3344	0	0.9169
-0.1187	-0.3344	-0.3044	0.7088
-0.1187	-0.3344	-0.3044	0.7088
-0.2339	-0.3344	-0.1226	0.8199
0.2555	-0.3344	-0.6302	0.9362
-0.1187	-0.3344	-0.3044	0.7088
0.2555	-0.3344	-0.6302	0.9362
-0.1187	-0.3344	-0.3044	0.7088
0.06341	-0.3344	-0.4855	0.8195
0.06341	-0.3344	-0.4855	0.8195
-0.1187	-0.3344	-0.3044	0.7088
-0.1187	-0.3344	-0.3044	0.7088
-0.2164	-0.3344	-0.1817	0.6494

U. NORMALIZING GROUP B C

NORMALIZED VALUES		ROW AVERAGE
-8.415E-1	5.240E-1	-1.587E-1
-5.240E-1	3.849E-1	-6.956E-2
-6.742E-1	8.415E-1	8.363E-2
-8.415E-1	3.849E-1	-2.283E-1
-1.282E0	1.254E-1	-5.782E-1
-1.036E0	6.742E-1	-1.811E-1
-5.240E-1	1.282E0	3.789E-1
-1.282E0	6.742E-1	-3.038E-1
-1.036E0	1.282E0	1.226E-1
-1.282E0	2.529E-1	-5.144E-1
-3.849E-1	1.645E0	6.302E-1
-1.282E0	1.282E0	0.000E0
-1.036E0	5.240E-1	-2.562E-1
-5.240E-1	1.645E0	5.606E-1
-1.036E0	1.036E0	0.000E0
-1.254E-1	1.645E0	7.599E-1
-5.240E-1	8.415E-1	1.587E-1
-8.415E-1	3.849E-1	-2.283E-1
-8.415E-1	5.240E-1	-1.587E-1
-1.036E0	8.415E-1	-9.749E-2
-1.036E0	1.010E-7	-5.182E-1
-1.645E0	2.529E-1	-6.961E-1
-1.282E0	2.529E-1	-5.144E-1
-1.645E0	5.240E-1	-5.606E-1
-1.282E0	3.849E-1	-4.484E-1
-1.645E0	2.529E-1	-6.961E-1
-1.282E0	5.240E-1	-3.789E-1
-1.645E0	1.645E0	-1.110E-16
-1.282E0	8.415E-1	-2.201E-1
-1.645E0	1.645E0	0.000E0
-1.645E0	6.742E-1	-4.855E-1
-1.645E0	8.415E-1	-4.019E-1
-1.645E0	8.415E-1	-4.019E-1
-1.282E0	1.645E0	1.817E-1

COLUMN AVERAGES	
-1.105	0.7977

GRAND AVERAGE
-0.1535

Scale Values of Group B C

$$S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$$

Scale Values	Grand Average	Row Average	(B/A(i))*.5
6.760E-2	-1.535E-1	-1.587E-1	1.393E0
-7.942E-3	-1.535E-1	-6.956E-2	2.093E0
-2.585E-1	-1.535E-1	8.363E-2	1.255E0
2.006E-1	-1.535E-1	-2.283E-1	1.551E0
6.282E-1	-1.535E-1	-5.782E-1	1.352E0
4.788E-2	-1.535E-1	-1.811E-1	1.112E0
-5.527E-1	-1.535E-1	3.789E-1	1.054E0
1.419E-1	-1.535E-1	-3.038E-1	9.727E-1
-2.542E-1	-1.535E-1	1.226E-1	8.207E-1
4.841E-1	-1.535E-1	-5.144E-1	1.240E0
-7.441E-1	-1.535E-1	6.302E-1	9.371E-1
-1.535E-1	-1.535E-1	0.000E0	7.421E-1
1.588E-1	-1.535E-1	-2.562E-1	1.219E0
-6.452E-1	-1.535E-1	5.606E-1	8.770E-1
-1.535E-1	-1.535E-1	0.000E0	9.178E-1
-9.701E-1	-1.535E-1	7.599E-1	1.074E0
-3.747E-1	-1.535E-1	1.587E-1	1.393E0
2.006E-1	-1.535E-1	-2.283E-1	1.551E0
6.760E-2	-1.535E-1	-1.587E-1	1.393E0
-5.479E-2	-1.535E-1	-9.749E-2	1.013E0
7.977E-1	-1.535E-1	-5.182E-1	1.836E0
5.442E-1	-1.535E-1	-6.961E-1	1.002E0
4.841E-1	-1.535E-1	-5.144E-1	1.240E0
3.381E-1	-1.535E-1	-5.606E-1	8.770E-1
3.583E-1	-1.535E-1	-4.484E-1	1.142E0
5.442E-1	-1.535E-1	-6.961E-1	1.002E0
2.456E-1	-1.535E-1	-3.789E-1	1.054E0
-1.535E-1	-1.535E-1	-1.110E-16	5.782E-1
4.370E-2	-1.535E-1	-2.201E-1	8.960E-1
-1.535E-1	-1.535E-1	0.000E0	5.782E-1
2.447E-1	-1.535E-1	-4.855E-1	8.202E-1
1.539E-1	-1.535E-1	-4.019E-1	7.651E-1
1.539E-1	-1.535E-1	-4.019E-1	7.651E-1
-2.717E-1	-1.535E-1	1.817E-1	6.500E-1

V. NORMALIZING GROUP C D

NORMALIZED VALUES		ROW AVERAGE
-0.6742	0.524	-0.07509
-0.8415	0.8415	0
-0.6742	1.645	0.4855
-1.282	0.2529	-0.5144
-0.8415	1.645	0.4019
-0.6742	1.282	0.3038
-0.6742	1.645	0.4855
-1.645	0.8415	-0.4019
0.2529	1.645	0.9491
-1.645	0.524	-0.5606
-0.8414	1.0101	-0.4207
-1.036	0.8415	-0.09749
-0.8415	1.645	0.4019
-1.645	0.1254	-0.7599
-1.282	0.8415	-0.2201
0.2529	1.282	0.7673
-0.8415	1.282	0.2201
-0.8415	1.645	0.4019

COLUMN AVERAGES	
-0.8764	1.0283

GRAND AVERAGE
0.07593

Scale Values of Group C D

$$S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$$

Scale Values	Grand Average	Row Average	(B/A(i))*.5
0.191	0.07591	-0.07509	1.571
0.07302	0.07591	0	1.119
-0.321	0.07591	0.4855	0.8116
0.704	0.07591	-0.5144	1.227
-0.2312	0.07591	0.4019	0.757
-0.2193	0.07591	0.3038	0.9624
-0.321	0.07591	0.4855	0.8116
0.3772	0.07591	-0.4019	0.757
-1.222	0.07591	0.9491	1.368
0.5595	0.07591	-0.5606	0.8678
1.0283	0.07591	-0.4207	2.2635
0.1707	0.07591	-0.09749	1.002
-0.2312	0.07591	0.4019	0.757
0.8809	0.07591	-0.7599	1.063
0.2682	0.07591	-0.2201	0.8866
-1.331	0.07591	0.7673	1.83
-0.1221	0.07591	0.2201	0.8866
-0.2312	0.07591	0.4019	0.757

W. INDEPENDENT VARIABLES WITH TRANSFORMED VALUES

The scale values, S_i, are adjusted to the desired scale using a linear transformation $Y = \alpha + \beta \times X, \ \beta > 0$.

No.	PER	AMMO	VEH	POL	Transformed Value
2	50	50	100	100	54.7229
4	50	100	50	25	54.7229
6	100	25	100	50	47.1713
8	50	50	50	75	55.4643
9	75	25	100	75	49.3552
11	100	50	50	100	60.6802
13	75	75	50	100	64.0568
16	75	100	50	100	63.1521
17	50	100	50	75	56.6995
18	100	50	100	50	70.9237
21	75	75	50	25	60.6067
22	75	50	50	25	54.3652
25	75	50	100	100	71.9459
26	100	75	50	50	65.7971
27	100	50	75	75	71.9459
30	50	100	100	25	63.3451
32	100	100	50	25	70.7587
33	50	100	50	100	58.3496
34	75	50	50	100	57.6611
37	50	50	50	50	47.1751
39	75	50	75	25	64.9312
40	50	75	50	50	57.3675
42	50	50	100	25	53.6024
43	75	75	75	25	74.5368
47	75	50	50	50	59.8433
50	100	50	50	25	54.7288
51	50	75	100	50	61.4969
61	50	50	75	100	53.1003
65	50	25	100	100	46.8027
73	75	25	100	50	49.4473
75	100	25	50	100	48.7059
76	75	25	75	50	50.5678
90	75	25	75	75	52.4799
95	100	25	75	75	54.1829
96	100	25	100	100	51.0699
97	100	25	75	50	53.1464
107	75	25	100	25	46.8027
120	100	25	75	25	48.8332
5	100	100	75	75	87.1829
7	100	100	50	100	73.4962
12	100	100	75	25	70.4419
28	75	50	100	75	69.7564
52	100	75	100	25	87.1498
58	75	75	100	25	77.6379
66	100	100	50	50	70.9840
71	100	50	100	75	71.6976
92	100	50	100	25	71.6976

108	100	75	75	25	79.3248
109	100	100	50	75	71.6976
112	75	100	75	25	76.2451
127	75	100	100	25	82.1945
1	75	25	50	50	46.1450
15	100	25	50	50	49.9997
23	75	25	75	25	52.2955
24	75	25	50	25	45.4832
36	50	50	50	25	44.7651
44	50	25	100	75	43.5990
45	50	25	100	50	43.5990
56	50	75	75	25	67.4624
57	50	100	75	25	78.3973
74	50	25	50	100	41.6600
79	50	25	50	75	40.7941
86	50	25	75	25	44.2505
87	50	25	50	50	40.7396
91	50	25	50	25	39.0695
98	100	25	100	25	41.6600
99	75	25	50	75	43.5533
101	50	25	75	50	43.5533
102	50	25	100	25	42.5425
103	75	25	100	100	46.8377
115	100	25	50	75	43.5533
126	75	25	50	100	46.8377
128	50	25	75	75	43.5533
131	100	25	100	75	45.1516
135	75	25	75	100	45.1516
138	100	25	50	25	43.5533
139	100	25	75	100	43.5533
143	50	25	75	100	42.6957
10	50	75	100	75	64.1305
19	50	100	75	50	63.2200
20	50	75	75	50	60.1997
29	50	100	100	50	65.7336
41	50	100	100	100	70.8868
49	50	50	100	75	63.8928
54	50	75	50	100	56.6540
60	75	50	100	25	65.0262
62	100	75	50	100	60.2518
64	100	50	75	25	69.1508
68	50	50	75	75	54.3472
72	100	75	50	25	61.4650
77	75	50	100	50	65.2300
78	50	75	50	75	55.5390
80	50	75	100	100	61.4650
81	50	100	50	50	51.6237
82	75	50	50	75	58.7995
83	50	75	75	100	65.7336
85	50	100	75	100	64.1305
89	100	50	50	50	62.6554
100	50	100	75	75	72.9300
104	75	100	50	25	69.8745
105	75	75	50	50	69.1508
111	75	75	50	75	67.3910
113	50	75	75	75	67.6347

116	50	100	100	75	69.8745
119	50	75	100	25	66.2760
121	100	75	50	75	61.4650
122	75	100	50	75	63.8424
123	75	50	75	50	61.4650
124	100	50	75	100	66.2648
132	75	50	75	75	65.1708
134	100	50	75	50	65.1708
140	100	50	50	75	60.0412
3	100	75	100	50	88.1615
14	75	75	100	75	86.4650
31	75	75	100	50	80.7985
35	75	100	100	100	95.5385
38	100	75	75	75	82.0901
46	75	75	75	75	82.2608
48	75	75	100	100	80.7985
53	75	100	75	50	90.8399
55	75	100	50	50	68.0122
59	75	100	100	50	93.4609
63	100	100	100	25	100.0000
69	100	75	75	100	87.8703
84	75	75	75	50	82.0901
93	75	100	100	75	98.0831
94	75	100	75	75	89.2717
110	100	50	100	100	66.2747
129	100	75	75	50	83.6583
133	75	75	75	100	82.0901
67	100	75	100	75	100.0000
70	50	50	50	100	50.0000
88	100	100	75	50	92.5000
106	100	100	75	100	95.5000
117	75	100	75	100	89.5000
118	100	100	100	100	105.0000
125	100	100	100	50	100.0000
142	100	100	100	75	103.0000
144	100	75	100	100	92.5000
114	75	50	75	100	72.9000
130	50	50	75	25	61.4000
136	50	75	50	25	59.2000
137	50	50	75	50	61.4000
141	50	50	100	50	70.6370

APPENDIX D. PAIRWISE INDEPENDENT VARIABLE COMPARISONS

The purpose of this appendix is to show that plotting each pair of independent variables results in elliptical shaped contour lines when the other two independent variables are held constant. The case of Ammunition plotted with Vehicles is explained below. The x and y axes refer to the independent variables being plotted while holding the other two independent variables constant. In Table 12 Ammunition is compared with Vehicles while holding Personnel and POL at 75%. The dependent variable's value is shown as a function of the independent variables in Fig. 8.

From this 3-D plot Fig. 9 reflects the contour plot where the cuts are made at dependent variable values of 50,60,70,80, and 90. These cuts are labeled in Fig. 9. The remaining pairs of independent variables are shown in Figures 10-21.

Table 12. DEPENDENT VARIABLE AS A FUNCTION OF AMMO AND VEH

	AMMO				
	***	25%	50%	75%	100%
VEH	50%	43.9	58.8	67.4	63.8
	75%	52.7	65.2	82.2	89.3
	100%	49.7	69.8	86.45	98.1

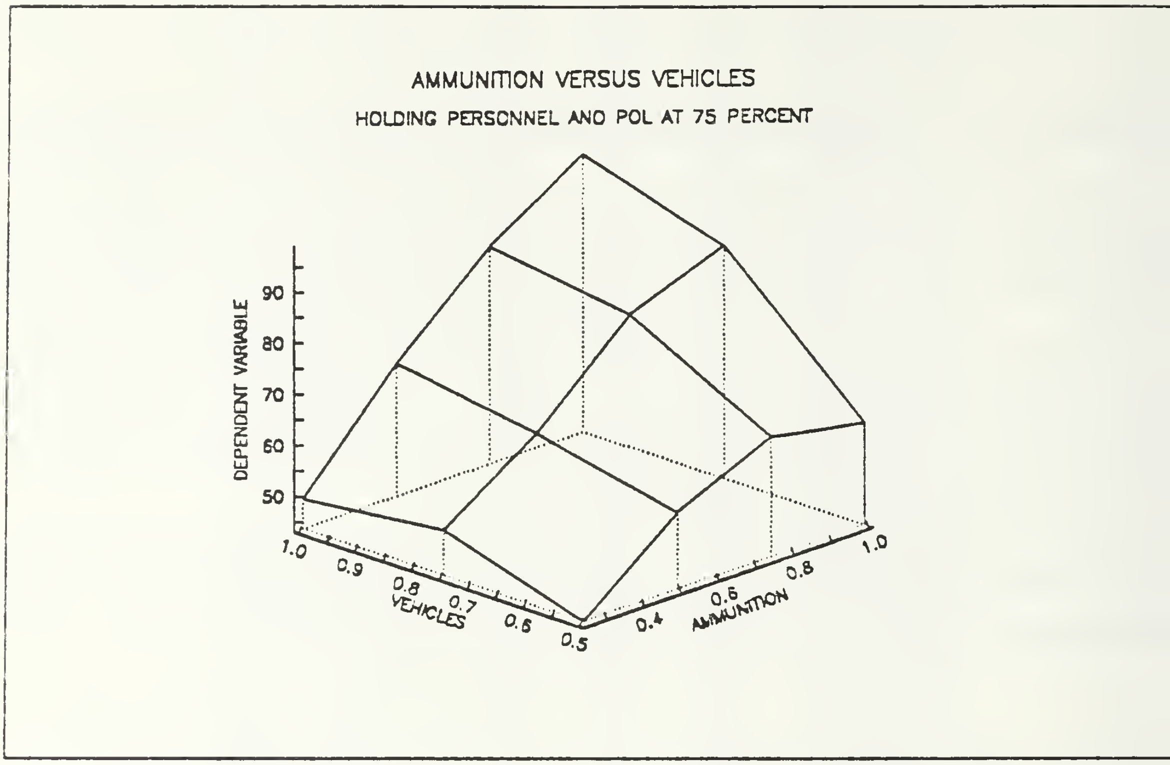


Figure 8. 3-D Contour Plot

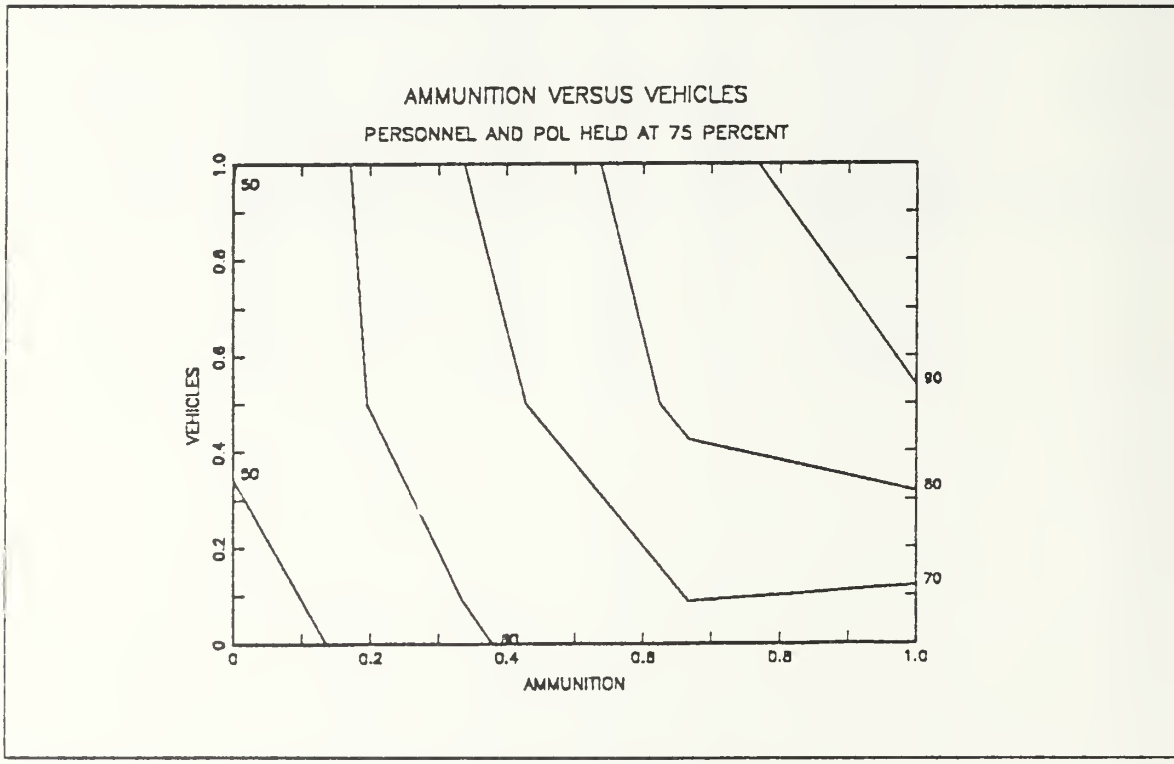


Figure 9. Ammunition Versus Vehicles Holding Others at 75%

A. PERSONNEL VERSUS AMMUNITION

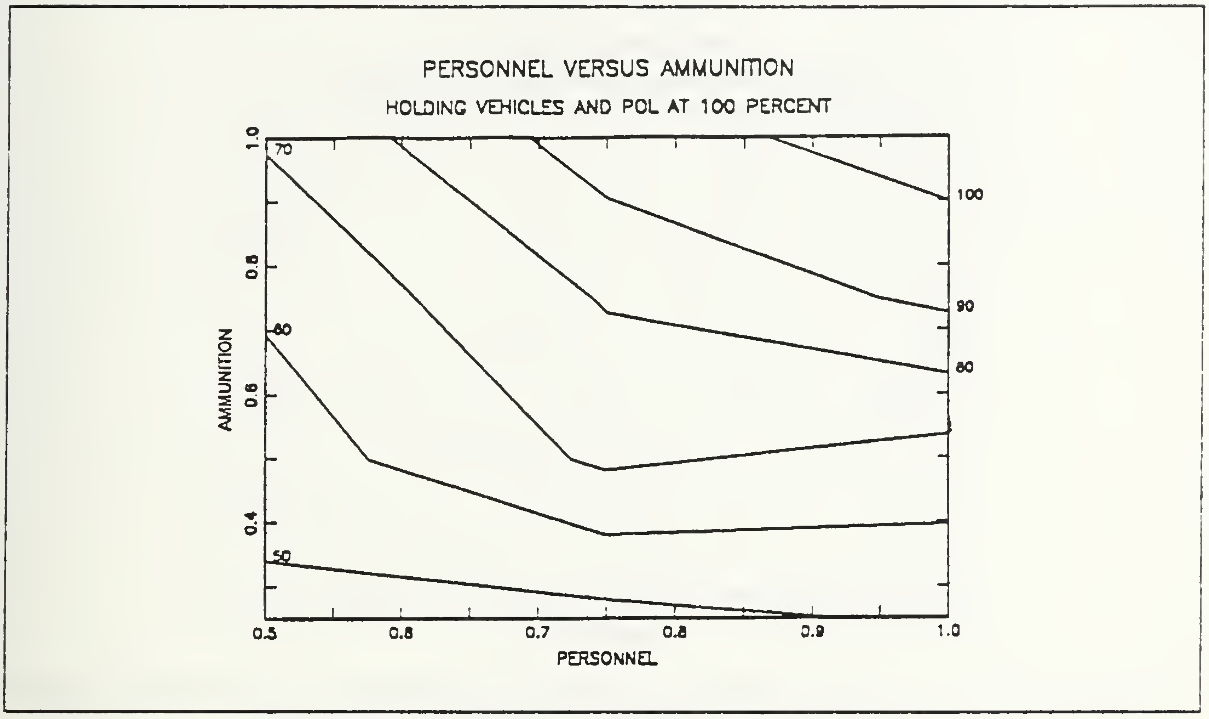


Figure 10. Personnel Versus Ammunition Holding Others at 100%

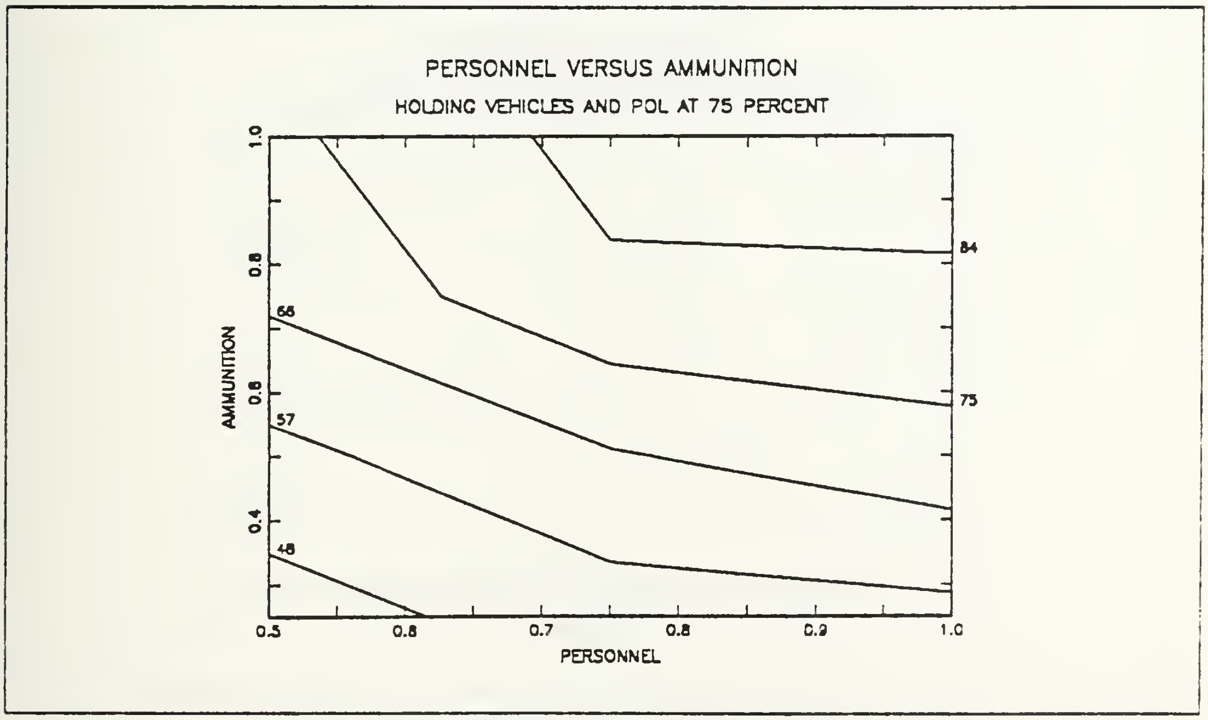


Figure 11. Personnel Versus Ammunition Holding Others at 75%

B. PERSONNEL VERSUS VEHICLES

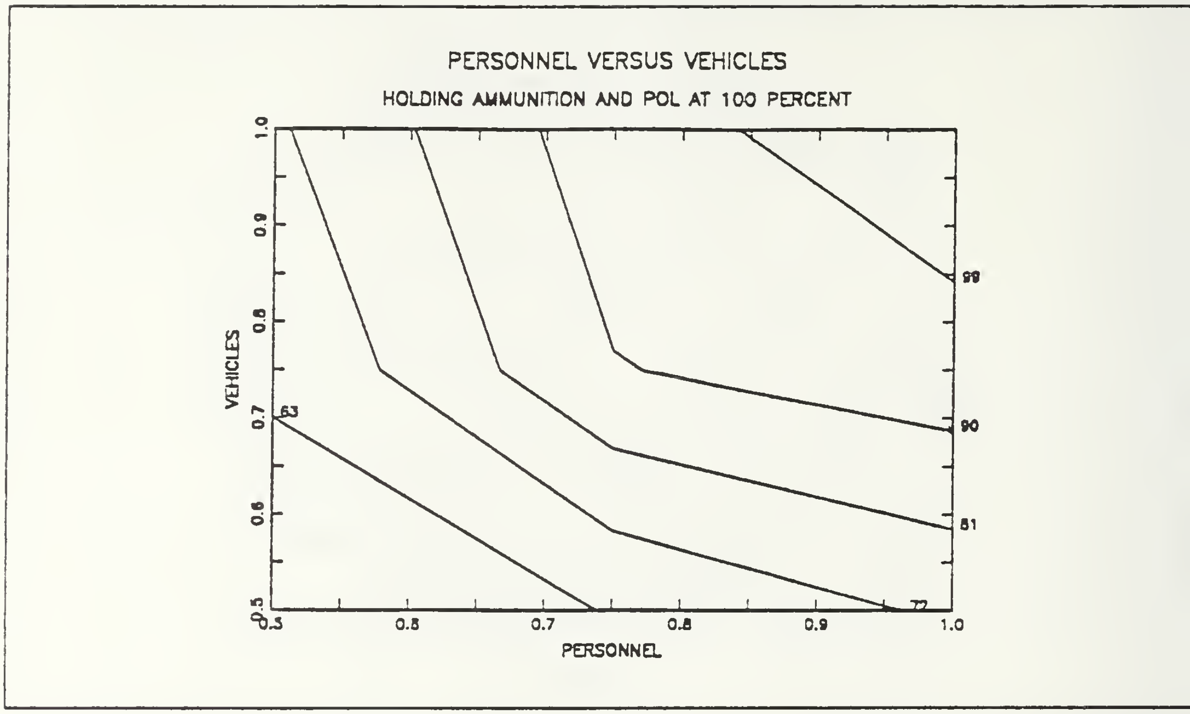


Figure 12. Personnel Versus Vehicles Holding Others at 100%

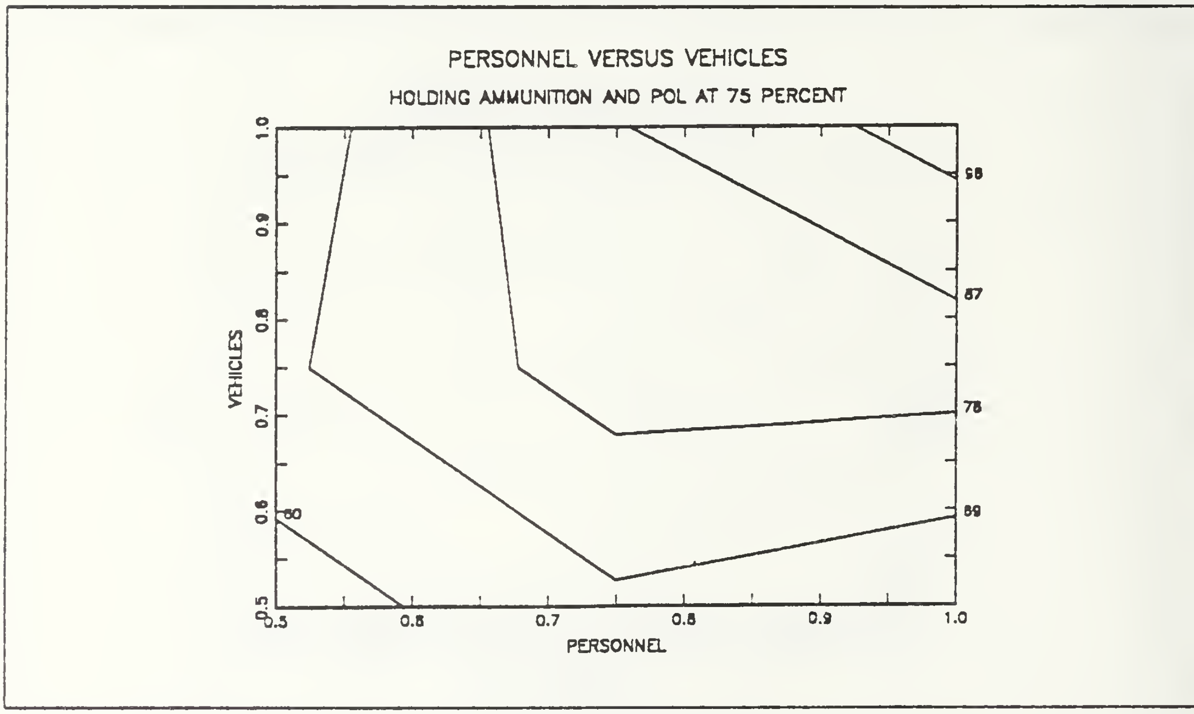


Figure 13. Personnel Versus Vehicles Holding Others at 75%

C. PERSONNEL VERSUS POL

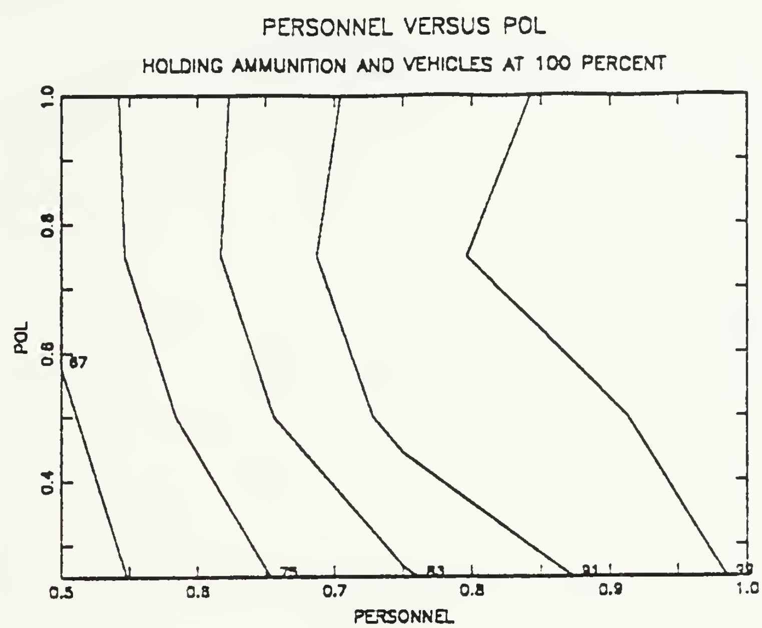


Figure 14. Personnel Versus POL Holding Others at 100%

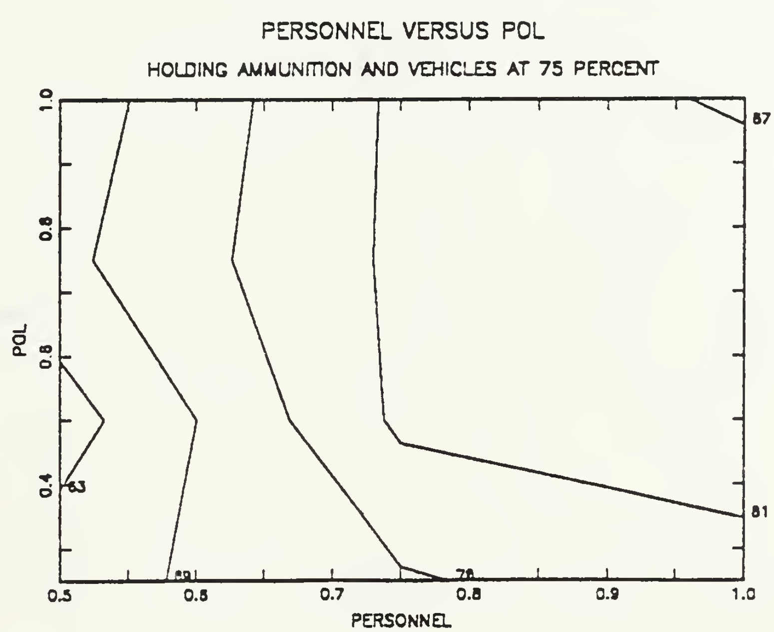


Figure 15. Personnel Versus POL Holding Others at 75%

D. AMMUNITION VERSUS VEHICLES

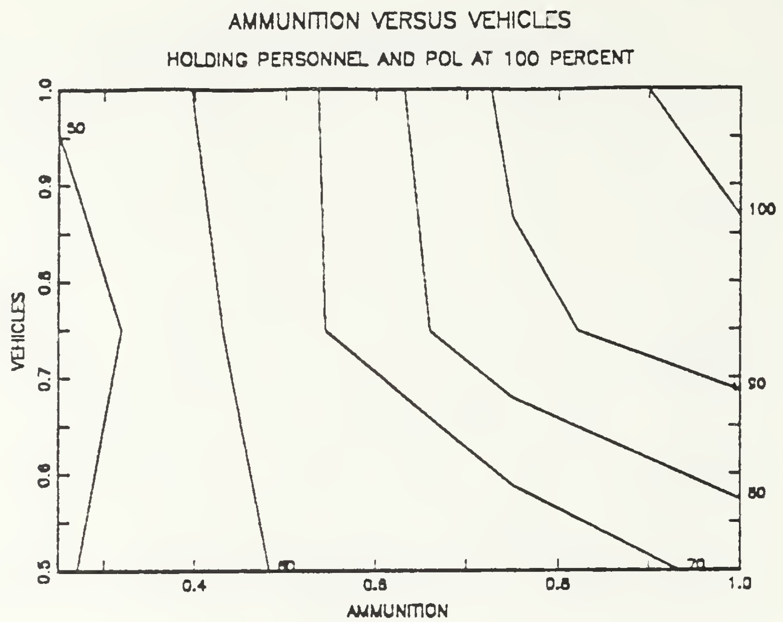


Figure 16. Ammunition Versus Vehicles Holding Others at 100%

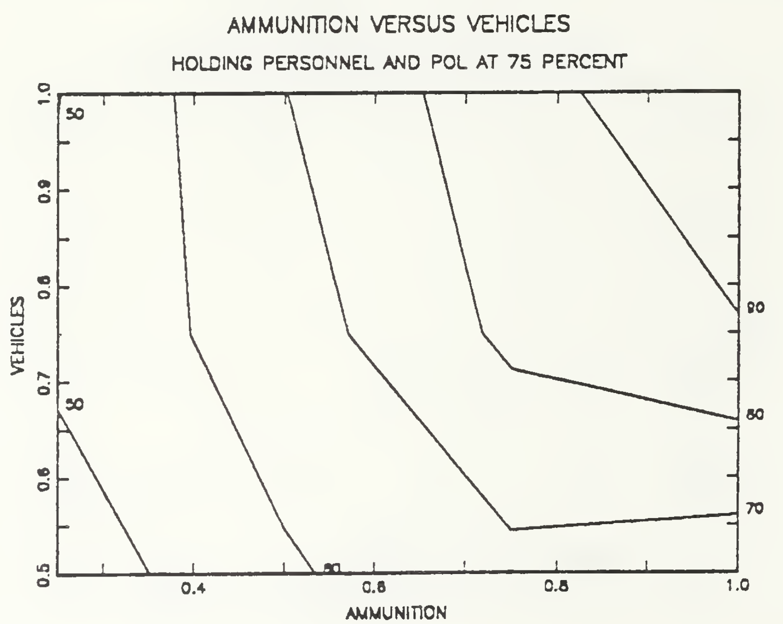


Figure 17. Ammunition Versus Vehicles Holding Others at 75%

E. AMMUNITION VERSUS POL

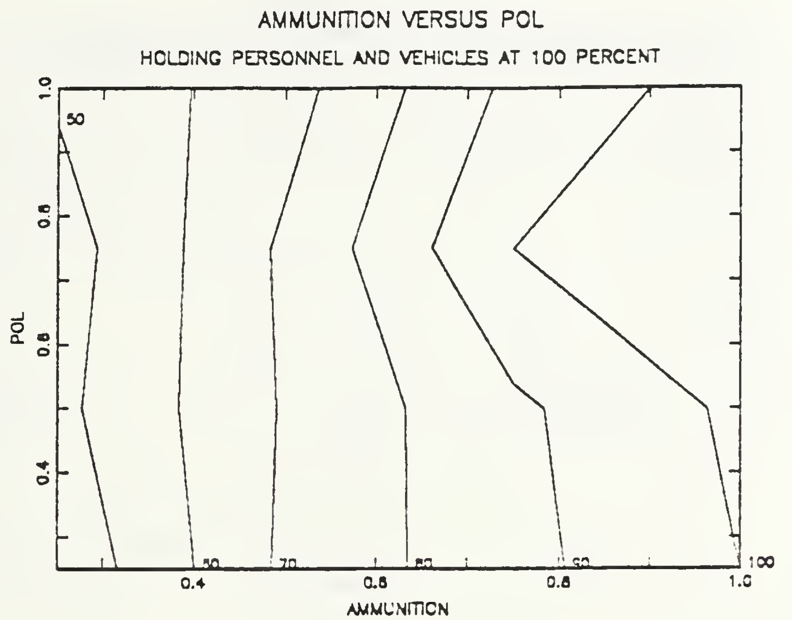


Figure 18. Ammunition Versus POL Holding Others at 100%

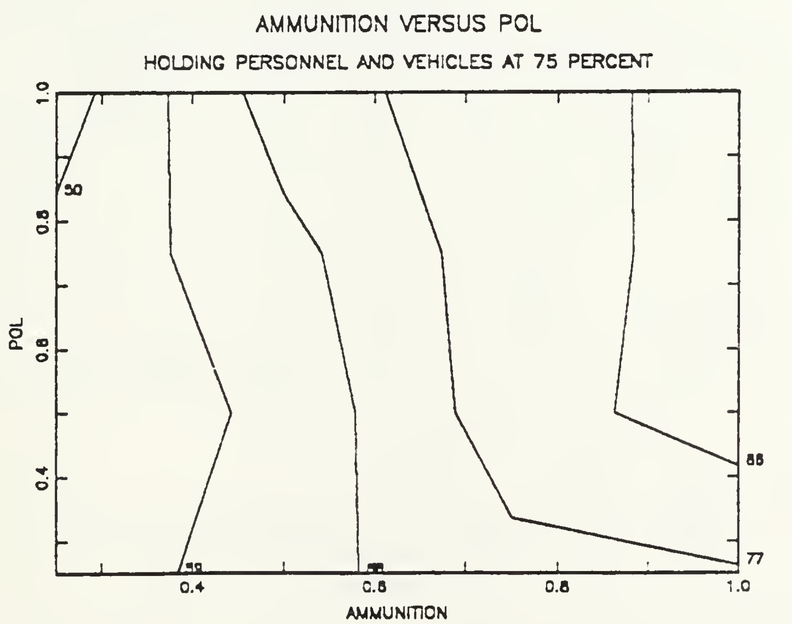


Figure 19. Ammunition Versus POL Holding Others at 75%

F. VEHICLES VERSUS POL

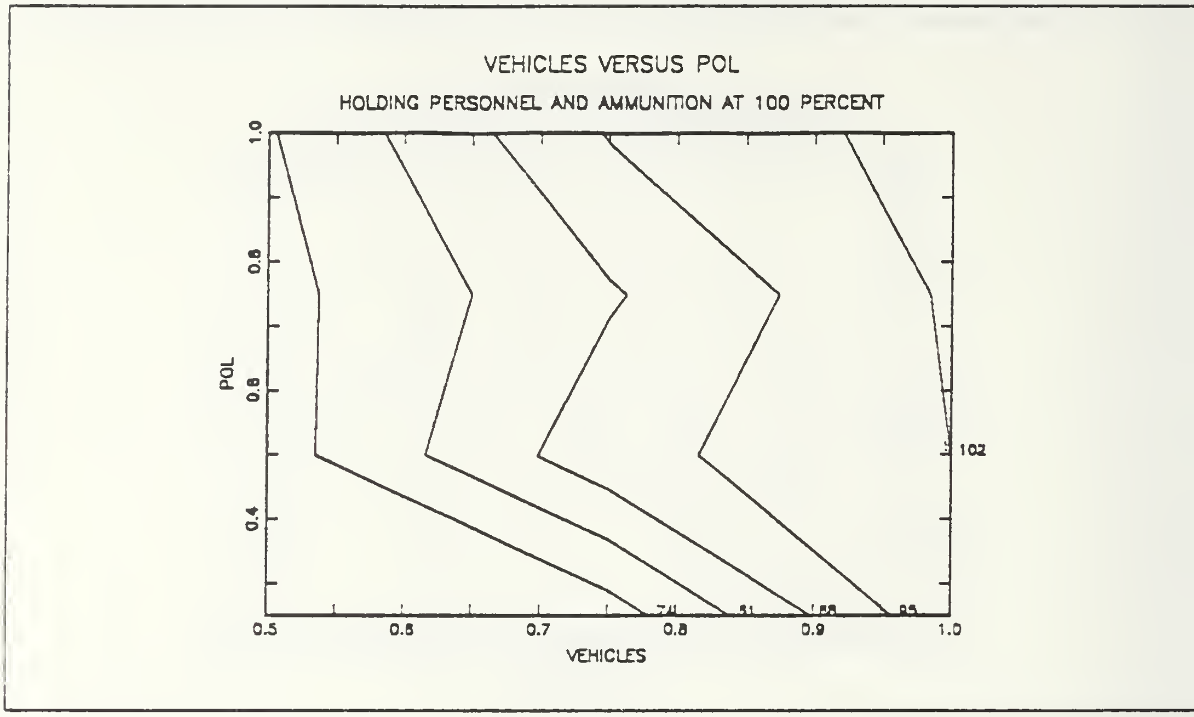


Figure 20. Vehicles Versus POL Holding Others at 100%

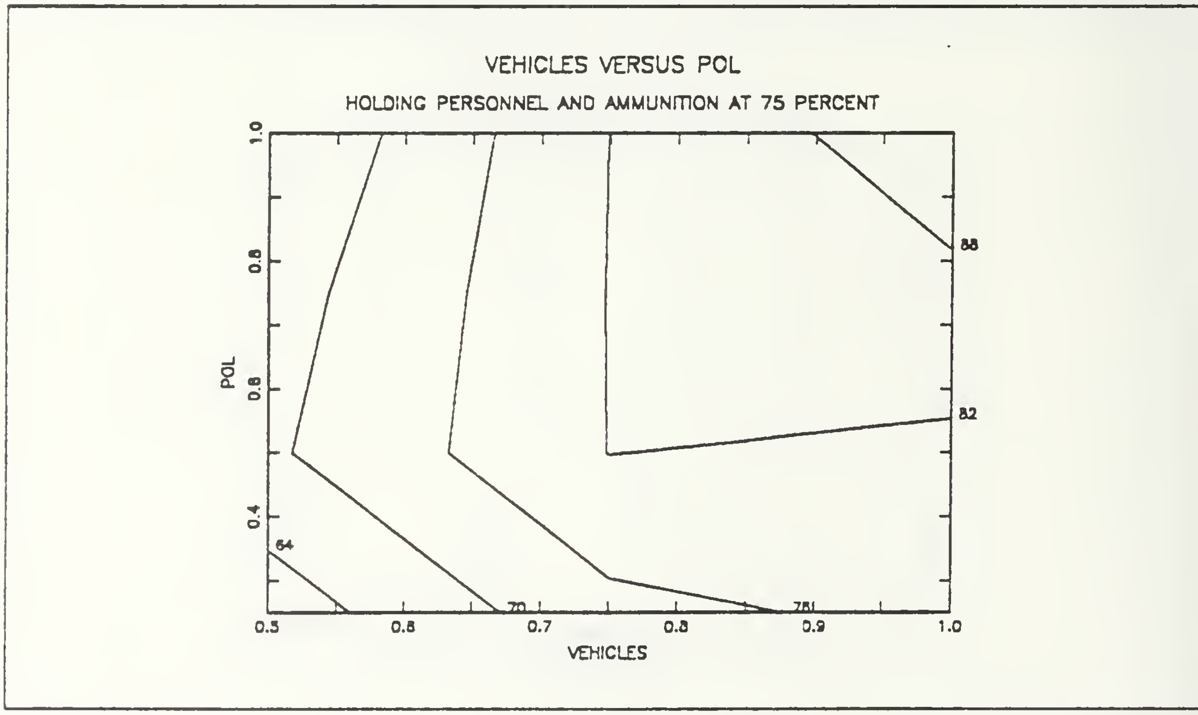


Figure 21. Vehicles Versus POL Holding Others at 75%

APPENDIX E. REGRESSION TECHNIQUES

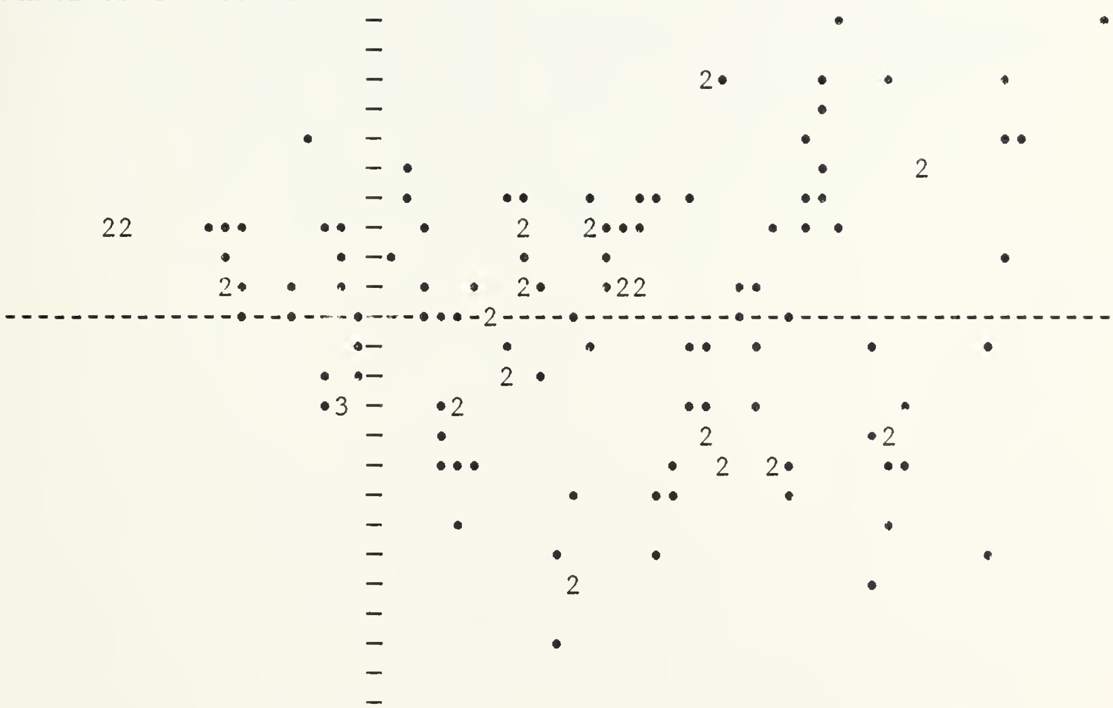
The results of regression of the values of unit effectiveness (YY) and the independent variables (PER, AMMO, VEH, POL) are shown in this appendix in an attempt to determine the best fitting function.

YY REGRESS PER, AMMO, VEH, POL

```
                              ANOVA

    SOURCE  DF     SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION   4        8.1838E4         2.0459E4         8.5905E1
  RESIDUAL 139        3.0961E4         2.3817E2
TOTAL      143        1.1280E5

R SQUARE:   0.785
STD ERROR:  15.4
   COEFFICIENTS    T STATISTICS
       -99.5119        -11.2074
          0.484          7.3031
         0.7982         16.5969
         0.4981          7.7018
         0.0375          0.7829
RANGE OF X:  -40 80
RANGE OF Y:  -60 40
```



YY REGRESS ln(PER), $\sqrt{AMMO}$, ln(VEH) , POL

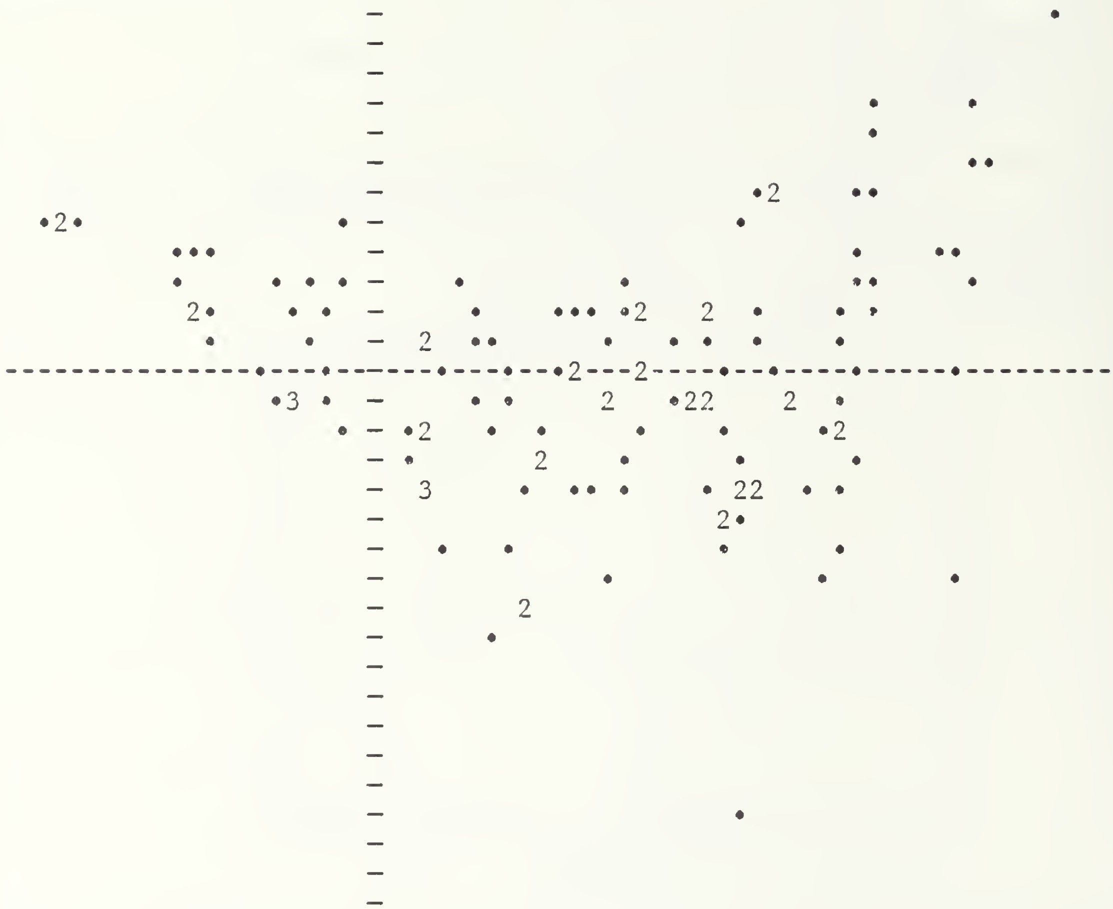
ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	8.8641E4	2.2160E4	1.1925E2
RESIDUAL	139	2.4158E4	1.8583E2	
TOTAL	143	1.1280E5		

R SQUARE: 0.786
STD ERROR: 13.6

COEFFICIENTS	T STATISTICS
537.5068	14.6404
35.4347	8.4483
87.0949	19.5124
155.2033	8.9433
8.9465	1.0335

RANGE OF X: 40 80
RANGE OF Y: 60 40



YY REGRESS ln(PER), AMMO, ln(VEH), POL

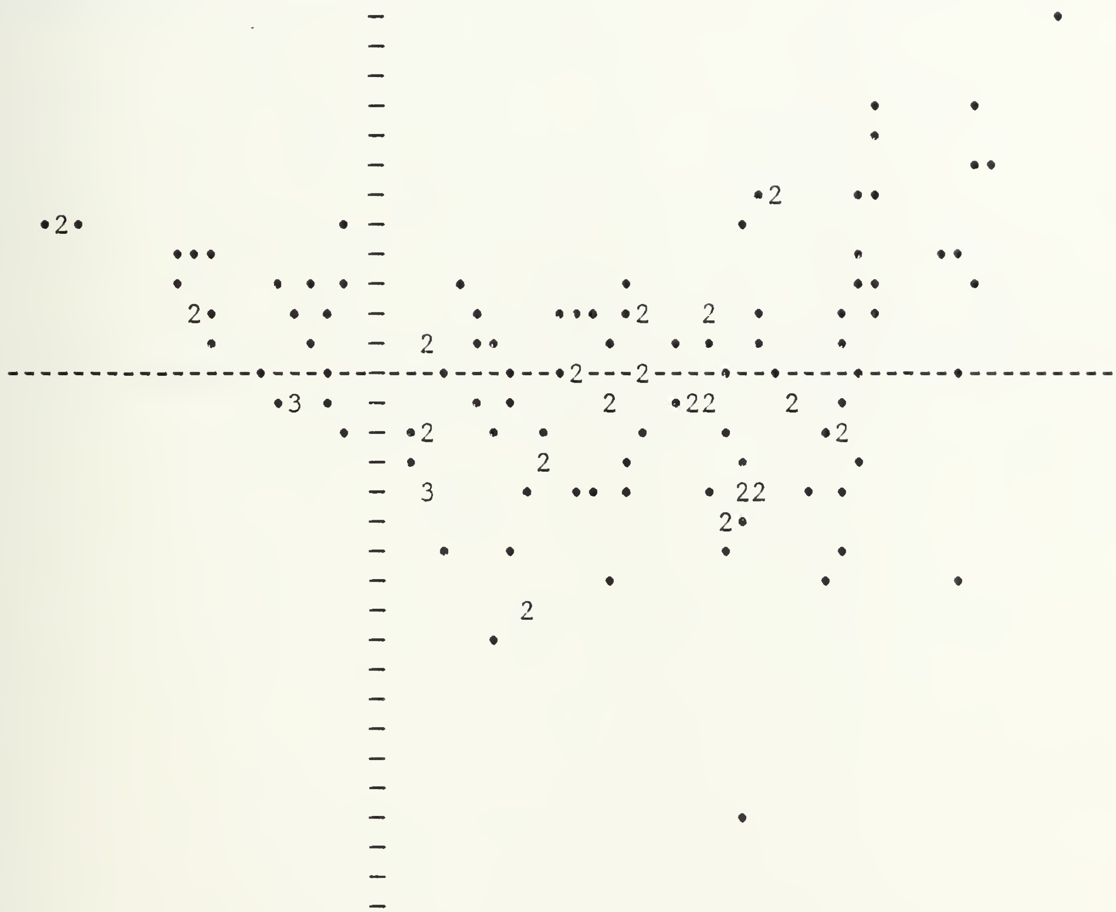
ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	8.8641E4	2.2160E4	1.1925E2
RESIDUAL	139	2.4158E4	1.8583E2	
TOTAL	143	1.1280E5		

R SQUARE: 0.786
STD ERROR: 13.6

COEFFICIENTS	T STATISTICS
537.5068	14.6404
35.4347	8.4483
43.5474	19.5124
155.2033	8.9433
8.9465	1.0335

RANGE OF X: 40 80
RANGE OF Y: 60 40



YY REGRESS ln(PER), AMMO, VEH, POL

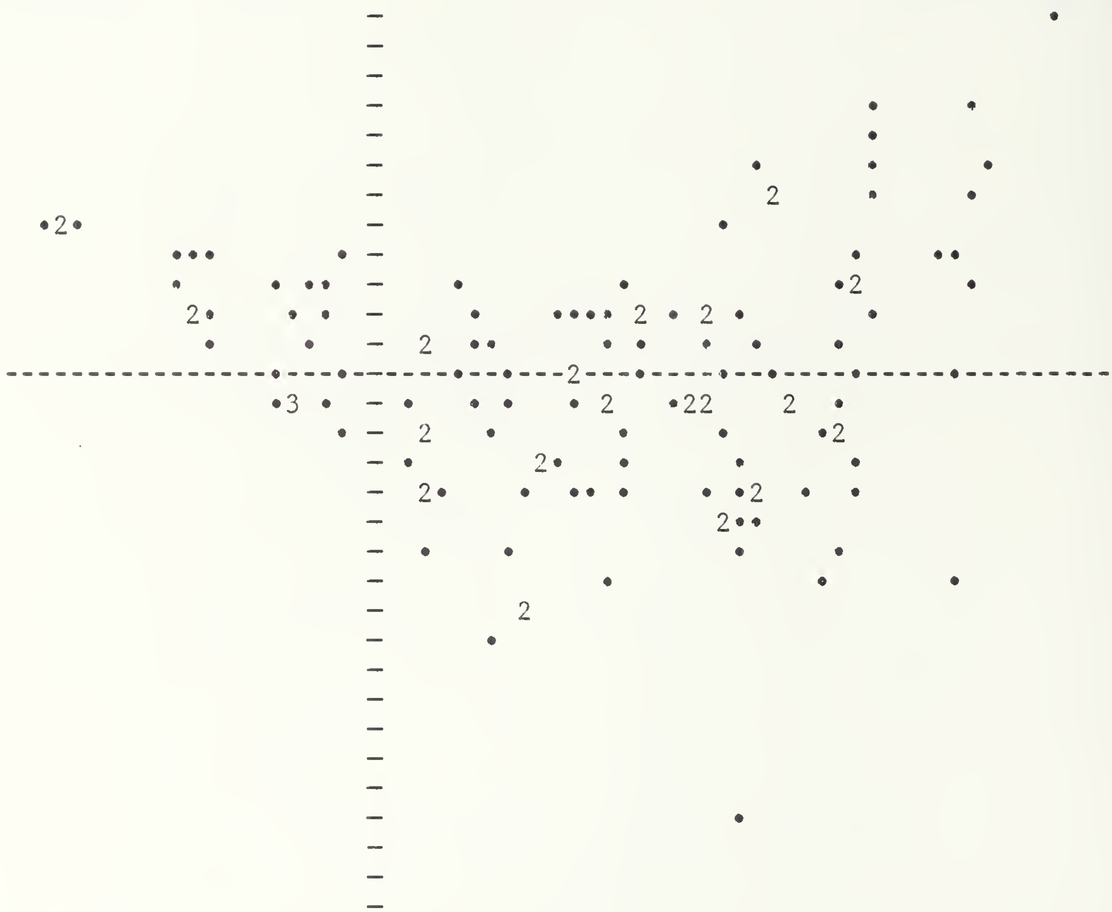
ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	8.8486E4	2.2122E4	1.1828E2
RESIDUAL	139	2.4313E4	1.8702E2	
TOTAL	143	1.1280E5		

R SQUARE: 0.784
STD ERROR: 13.7

COEFFICIENTS	T STATISTICS
464.8335	15.2787
35.4387	8.422
43.5224	19.4403
36.4316	8.8687
2.2767	1.0071

RANGE OF X: 40 80
RANGE OF Y: 60 40



YY REGRESS $PER^{1.5}$, $AMMO^{0.0125}$, ln(VEH), POL

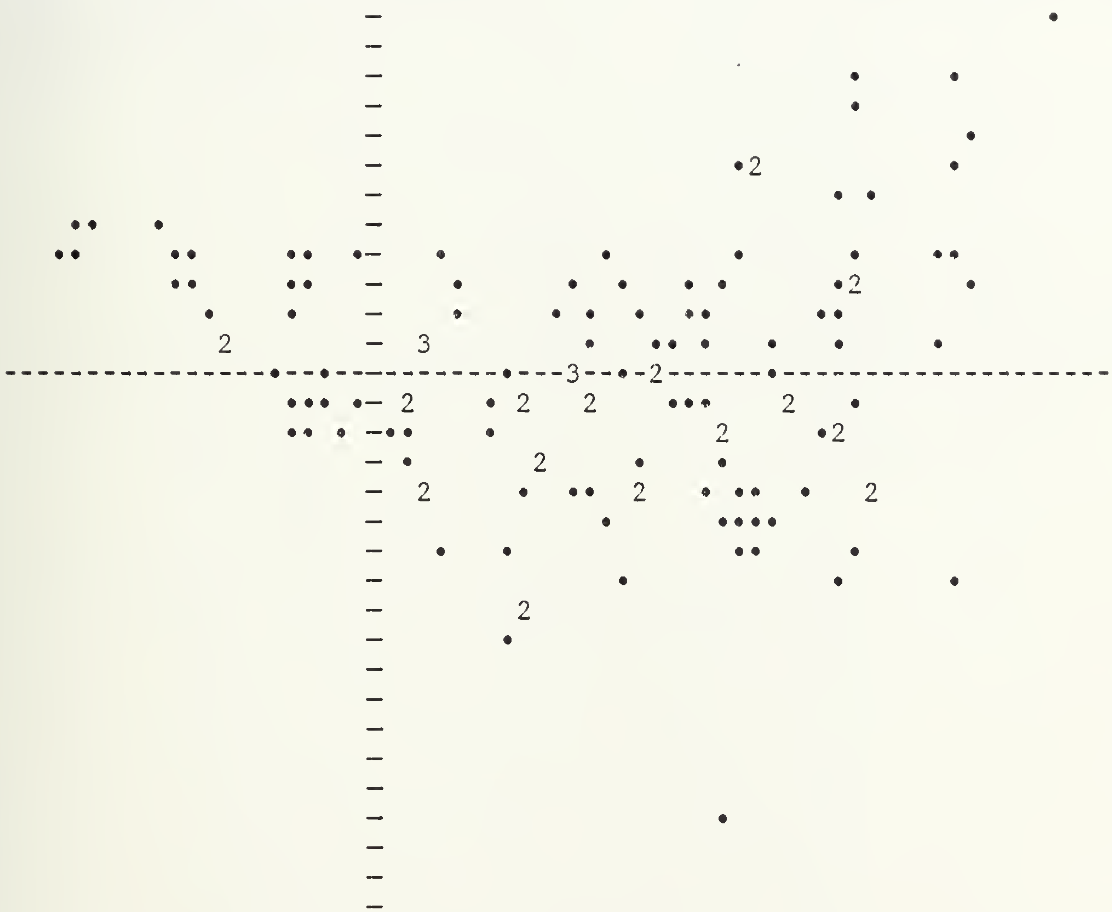
ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	8.6664E4	2.1666E4	1.0777E2
RESIDUAL	139	2.6135E4	2.0104E2	
TOTAL	143	1.1280E5		

R SQUARE: 0.768
STD ERROR: 14.2

COEFFICIENTS	T STATISTICS
3648.21	19.288
0.0355	7.5558
3315.9122	18.7275
36.3996	8.5433
2.167	0.9246

RANGE OF X: 40 80
RANGE OF Y: 60 40



YY REGRESS $PER^{0.9}$, AMMO, $VEH^{0.9}$, POL

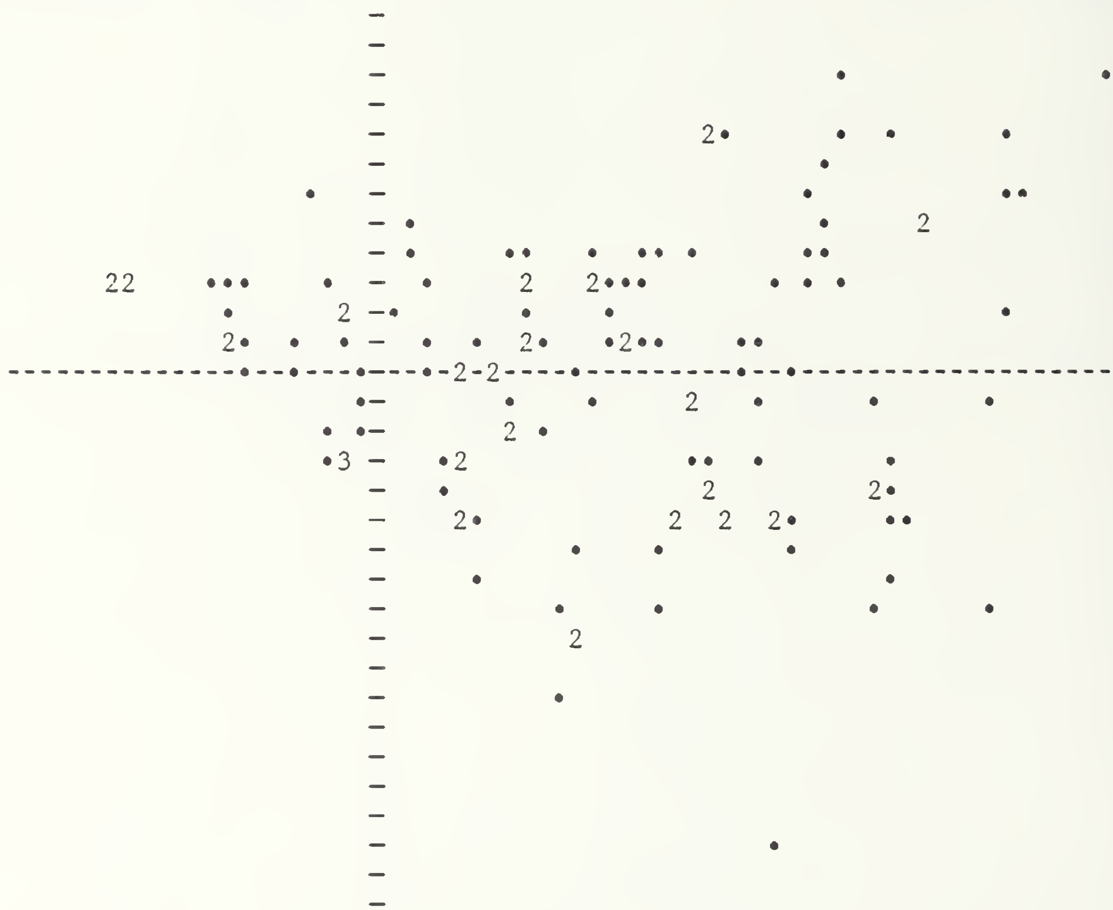
ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	8.2056E4	2.0514E4	8.6744E1
RESIDUAL	139	3.0744E4	2.3649E2	
TOTAL	143	1.1280E5		

R SQUARE: 0.727
STD ERROR: 15.4

COEFFICIENTS	T STATISTICS
107.5996	11.3162
0.8306	7.3665
0.7984	16.6619
0.8531	7.7538
0.0377	0.7901

RANGE OF X: 40 80
RANGE OF Y: 60 40



YY REGRESS $\sqrt{PER}$, AMMO, $\sqrt{VEH}$, POL

ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	8.2886E4	2.0722E4	9.0055E1
RESIDUAL	139	2.9913E4	2.3010E2	
TOTAL	143	1.1280E5		

R SQUARE: 0.735
STD ERROR: 15.2

COEFFICIENTS	T STATISTICS
-172.3749	-11.5384
8.4535	7.6123
0.7994	16.9146
8.6144	7.9528
0.0385	0.8186

DURBIN-WATSON: 1.47
RANGE OF X: 20 100
RANGE OF Y: 20 20

APPENDIX F. BEST FIT REGRESSION MODEL

Regression of the values of unit effectiveness (YY) and the independent variables (PER, AMMO, VEH, POL) was conducted resulting in the following best fitting model.

$X1 = (PER - 100)^2$, $X2 = (AMMO - 100)^2$

$X3 = (VEH - 100)^2$, $X4 = (POL - 100)^2$

```
YY  REGRESS  X1, X2, X3, X4
```

ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	3.0603E4	7.6509E3	1.8601E2
RESIDUAL	139	5.7173E3	4.1132E1	
TOTAL	143	3.6321E4		

R SQUARE: 0.842589
STD ERROR: 6.41338

COEFFICIENTS	T STATISTICS
88.9781	75.0341
-0.0056	-11.0731
-0.0055	-22.3976
-0.0054	-10.7318
-0.0005	-2.1451

RANGE OF X: 20 100
RANGE OF Y: 15 20

APPENDIX G. WILCOXON SIGN RANK TEST

The transformed values are represented by X and the predicted values are represented by Y. The Wilcoxon Sign Rank Test for Location of Paired Samples tests the hypothesis that the median difference between X and Y is zero.

X	Y	DIF (x - y)	|DIF|	DIF |RANK|	DIF |SIGN RANK|
54.72	61.23	-6.505	6.505	99	-99
54.72	58.67	-3.943	3.943	69	-69
47.17	56.79	-9.619	9.619	123	-123
55.46	47.42	8.049	8.049	114	114
49.36	54.23	-4.873	4.873	85	-85
60.68	61.73	-1.048	1.048	24	-24
64.06	68.54	-4.484	4.484	77	-77
63.15	71.98	-8.826	8.826	119	-119
56.70	61.17	-4.466	4.466	76	-76
70.92	73.98	-3.054	3.054	56	-56
60.61	65.73	-5.121	5.121	88	-88
54.37	55.42	-1.050	1.050	25	-25
71.95	71.73	0.218	0.218	7	7
65.80	70.79	-4.993	4.993	87	-87
71.95	71.54	0.405	0.405	11	11
63.35	72.17	-8.820	8.820	118	-118
70.76	72.67	-1.907	1.907	38	-38
58.35	61.48	-3.128	3.128	57	-57
57.66	58.23	-0.567	0.567	13	-13
47.18	46.48	0.697	0.697	18	18
64.93	65.54	-0.609	0.609	15	-15
57.37	56.79	0.577	0.577	14	14
53.60	58.42	-4.813	4.813	84	-84
74.54	75.85	-1.316	1.316	28	-28
59.84	56.98	2.865	2.865	51	51
54.73	58.92	-4.187	4.187	74	-74
61.50	70.29	-8.794	8.794	117	-117
53.10	57.85	-4.753	4.753	81	-81
46.80	44.04	2.762	2.762	49	49
49.45	53.29	-3.843	3.843	64	-64
48.71	44.54	4.165	4.165	73	73
50.57	49.92	0.652	0.652	16	16
52.48	50.85	1.627	1.627	32	32
54.18	54.35	-0.170	0.170	6	-6
51.07	58.04	-6.971	6.971	103	-103
53.15	53.42	-0.269	0.269	10	-10
46.80	51.73	-4.925	4.925	86	-86
48.83	51.85	-3.020	3.020	54	-54
87.18	85.29	1.892	1.892	37	37
73.50	75.48	-1.982	1.982	39	-39
70.44	82.79	-12.350	12.350	137	-137
69.76	71.42	-1.659	1.659	34	-34

87.15	82.73	4.422	4.422	75	75
77.64	79.23	-1.590	1.590	31	-31
70.98	74.23	-3.244	3.244	61	-61
71.70	74.92	-3.218	3.218	60	-60
71.70	72.42	-0.718	0.718	20	-20
79.32	79.35	-0.028	0.028	2	-2
71.70	75.17	-3.468	3.468	63	-63
76.25	79.29	-3.045	3.045	55	-55
82.19	82.67	-0.471	0.471	12	-12
46.14	39.79	6.354	6.354	96	96
50.00	43.29	6.709	6.709	100	100
52.30	48.35	3.942	3.942	68	68
45.48	38.23	7.255	7.255	106	106
44.77	44.92	-0.151	0.151	5	-5
43.60	43.73	-0.129	0.129	4	-4
43.60	42.79	0.808	0.808	21	21
67.46	65.35	2.109	2.109	41	41
78.40	68.79	9.607	9.607	122	122
41.66	30.54	11.120	11.120	132	132
40.79	30.23	10.570	10.570	129	129
44.25	37.85	6.397	6.397	98	98
40.74	29.29	11.450	11.450	134	134
39.07	27.73	11.340	11.340	133	133
41.66	55.23	-13.570	13.570	140	-140
43.55	40.73	2.825	2.825	50	50
43.55	39.42	4.138	4.138	72	72
42.54	41.23	1.314	1.314	27	27
46.84	54.54	-7.703	7.703	110	-110
43.55	44.23	-0.675	0.675	17	-17
46.84	41.04	5.797	5.797	94	94
43.55	40.35	3.200	3.200	59	59
45.15	57.73	-12.580	12.580	138	-138
45.15	51.17	-6.014	6.014	95	-95
43.55	41.73	1.825	1.825	35	35
43.55	54.67	-11.110	11.110	131	-131
42.70	40.67	2.030	2.030	40	40
64.13	71.23	-7.098	7.098	104	-104
63.22	70.35	-7.133	7.133	105	-105
60.20	66.92	-6.716	6.716	101	-101
65.73	73.73	-7.995	7.995	113	-113
70.89	74.98	-4.091	4.091	71	-71
63.89	60.92	2.977	2.977	53	53
56.65	58.04	-1.387	1.387	30	-30
65.03	68.92	-3.889	3.889	66	-66
60.25	72.04	-11.790	11.790	135	-135
69.15	69.04	0.110	0.110	3	3
54.35	57.54	-3.193	3.193	58	-58
61.47	69.23	-7.763	7.763	111	-111
65.23	70.48	-5.248	5.248	89	-89
55.54	57.73	-2.189	2.189	43	-43
61.47	71.54	-10.080	10.080	127	-127
51.62	60.23	-8.604	8.604	116	-116
58.80	57.92	0.884	0.884	23	23
65.73	68.17	-2.432	2.432	45	-45
64.13	71.60	-7.473	7.473	108	-108
62.66	60.48	2.177	2.177	42	42

72.93	71.29	1.639	1.639	33	33
69.87	69.17	0.709	0.709	19	19
69.15	67.29	1.860	1.860	36	36
67.39	68.23	-0.837	0.837	22	-22
67.63	67.85	-0.218	0.218	8	-8
69.87	74.67	-4.791	4.791	82	-82
66.28	68.73	-2.452	2.452	46	-46
61.47	71.73	-10.260	10.260	128	-128
63.84	71.67	-7.823	7.823	112	-112
61.47	67.10	-5.638	5.638	92	-92
66.26	71.85	-5.588	5.588	91	-91
65.17	68.04	-2.870	2.870	52	-52
65.17	70.60	-5.432	5.432	90	-90
60.04	61.42	-1.374	1.374	29	-29
88.16	84.29	3.871	3.871	65	65
86.47	81.73	4.737	4.737	80	80
80.80	80.79	0.008	0.008	1	1
95.54	85.48	10.060	10.060	126	126
82.09	81.85	0.237	0.237	9	9
82.26	78.35	3.908	3.908	67	67
80.8	82.04	-1.242	1.242	26	-26
90.84	80.85	9.987	9.987	125	125
68.01	70.73	-2.716	2.716	47	-47
93.46	84.23	9.233	9.233	121	121
100.00	86.17	13.830	13.830	141	141
87.87	82.17	5.705	5.705	93	93
82.09	77.42	4.675	4.675	79	79
98.08	85.17	12.920	12.920	139	139
89.27	81.79	7.481	7.481	109	109
66.27	75.23	-8.953	8.953	120	-120
83.66	80.92	2.743	2.743	48	48
82.09	78.67	3.425	3.425	62	62
100.00	85.23	14.770	14.770	143	143
50.00	47.73	2.272	2.272	44	44
92.50	84.35	8.147	8.147	115	115
95.50	85.60	9.897	9.897	124	124
89.50	82.10	7.397	7.397	107	107
105.00	88.98	16.020	16.020	144	144
100.00	87.73	12.270	12.270	136	136
103.00	88.67	14.330	14.330	142	142
92.50	85.54	6.959	6.959	102	102
72.90	68.35	4.547	4.547	78	78
61.40	55.04	6.359	6.359	97	97
59.20	55.23	3.972	3.972	70	70
61.40	56.60	4.797	4.797	83	83
70.64	59.98	10.660	10.660	130	130

T PLUS = 5043 T MINUS = 5397

Z PLUS = -0.354 Z MINUS = 0.352

P-VALUE (RIGHT-TAIL) FROM Z PLUS = 0.3617

P-VALUE (RIGHT-TAIL) FROM Z MINUS = 0.3624

APPENDIX H. BLUE VERUS RED BATTLE

This appendix presents a battle between Blue and Red forces. Section A is a battle with no resupply or reinforcements, and Section B is a battle with the Blue receiving reinforcements by time, t = 30 minutes.

A. BLUE VERSUS RED BATTLE WITH NO RESUPPLY

Time (min)	Blue DPF	Red DPF	Blue PABIP	Red PABIP	Blue SIP	Red SIP	(Red - Blue) SIP's
0	1.0000	1.0000	1000	1500	1000	100	-900.0
2	1.0000	1.0000	1000	1500	1000	122.1	-877.9
4	0.9999	1.0000	999.9	1500	999.9	149.1	-850.8
6	0.9997	0.9999	999.7	1500	999.7	182.0	-817.7
8	0.9995	0.9998	999.5	1500	999.5	222.3	-777.2
10	0.9992	0.9997	999.2	1500	999.2	271.4	-727.8
12	0.9988	0.9996	998.8	1499	998.8	331.3	-667.5
14	0.9984	0.9994	998.3	1499	998.4	404.5	-593.9
16	0.9979	0.9992	997.8	1499	997.9	493.8	-504.1
18	0.9973	0.9990	997.3	1499	997.3	602.8	-394.4
20	0.9967	0.9988	996.6	1498	996.7	735.9	-260.7
22	0.9949	0.9986	994.9	1498	994.9	898.4	-96.5
24	0.9923	0.9983	992.3	1497	992.3	1097.0	104.4
26	0.9888	0.9980	988.9	1497	988.8	1339.0	349.9
28	0.9844	0.9977	984.5	1496	984.4	1634.0	649.7
30	0.9791	0.9974	979.3	1496	979.1	1995	1016.0
32	0.9673	0.9901	968.9	1482	967.3	1980	1013.0
34	0.9521	0.9768	955.7	1457	952.1	1954	1002.0
36	0.9335	0.9576	939.6	1420	933.5	1915	981.7
38	0.9115	0.9324	920.6	1372	911.5	1865	953.3
40	0.8862	0.9013	898.7	1312	886.2	1803	916.4
42	0.8575	0.8642	874	1240	857.5	1728	870.9
44	0.8255	0.8212	846.5	1157	825.5	1642	816.9
46	0.7901	0.7722	816	1062	790.1	1544	754.3
48	0.7513	0.7173	782.7	955	751.3	1435	683.2
50	0.7092	0.6563	746.5	836.6	709.2	1313	603.5
52	0.6637	0.5895	707.5	706.7	663.7	1179	515.3
54	0.6148	0.5166	665.6	565.1	614.8	1033	418.5
56	0.5626	0.4379	620.8	411.8	562.6	875.7	313.2
58	0.507	0.3531	573.1	246.9	507	706.3	199.3
60	0.448	0.2624	522.6	70.4	448	524.8	76.8

B. BLUE VERSUS RED BATTLE WITH REINFORCEMENTS

Blue forces receive reinforcements by time, t = 30 minutes which is reflected by an increase in the Blue forces PABIP, and SIP.

Time (min)	Blue DPF	Red DPF	Blue PABIP	Red PABIP	Blue SIP	Red SIP	(Red - Blue) SIP's
0	1.0000	1.0000	1050	2000	1050	100.0	-950.0
2	1.0000	1.0000	1061	2000	1061	122.1	-938.9
4	0.9999	1.0000	1074	2000	1074	149.1	-925.3
6	0.9997	0.9999	1091	2000	1091	182.0	-908.7
8	0.9995	0.9998	1111	2000	1111	222.3	-888.3
10	0.9992	0.9997	1135	1999	1135	271.0	-863.4
12	0.9988	0.9996	1164	1999	1164	331.3	-833.0
14	0.9984	0.9994	1200	1999	1200	404.5	-795.9
16	0.9979	0.9992	1244	1998	1244	493.8	-750.5
18	0.9973	0.999	1298	1998	1298	602.8	-695.2
20	0.9967	0.9988	1364	1998	1364	735.9	-627.7
22	0.9949	0.9986	1443	1997	1443	898.4	-544.3
24	0.9923	0.9983	1539	1997	1539	1097.0	-441.9
26	0.9888	0.998	1655	1996	1655	1339.0	-316.4
28	0.9844	0.9977	1797	1995	1797	1634.0	-163.0
30	0.9791	0.9974	1970	1995	1970	1995.0	-24.3
32	0.9673	0.9901	1935	1980	1935	1980.0	45.0
34	0.9521	0.9768	1904	1954	1904	1954.0	50.0
36	0.9335	0.9576	1867	1915	1867	1915.0	48.0
38	0.9115	0.9324	1823	1865	1823	1865.0	42.0
40	0.8862	0.9013	1772	1803	1772	1803.0	31.0
42	0.8575	0.8642	1715	1728	1715	1728.0	13.0
44	0.8255	0.8212	1651	1642	1651	1642.0	-8.6
46	0.7901	0.7722	1580	1544	1580	1544.0	-35.8
48	0.7513	0.7173	1503	1435	1503	1435.0	-68.1
50	0.7092	0.6563	1418	1313	1418	1313.0	-105.7
52	0.6637	0.5895	1327	1179	1327	1179.0	-148.4
54	0.6148	0.5166	1230	1033	1230	1033.0	-196.3
56	0.5626	0.4379	1125	875.7	1125	875.7	-249.4
58	0.5070	0.3531	1014	706.3	1014	706.3	-307.7
60	0.4480	0.2624	896	524.9	896	524.9	-371.1

APPENDIX I. PROGRAMS DEVELOPED FOR THESIS

This appendix contains the A Programming Language (APL) programs used in this thesis. The following programs are available at the Naval Postgraduate School.

A. STANDARD NORMAL TABLE

```
      ∇GAUSS[⎕]∇
      ∇ Z←GAUSS X;T;A;B;B
[1]   T←(1+(|X)×0.2316419)*¯1
[2]   B← 0.31938153 ¯0.356563782 1.781477937 ¯1.821255978 1.330274429
[3]   A←,(T∘.* 1 2 3 4 5)+.×B
[4]   B←A×(*(-((X*2)÷2)))÷(○2)*0.5
[5]   Z←((X<0)×B)+(X≥0)×(1-B)
      ∇
```

B. STANDARD NORMAL QUANTILE

```
      ∇NQUAN[⎕]∇
      ∇ Z←NQUAN P;A;B;C;D
[1]   →((+/(A←(P≤0)∨(P≥1)))>0)/L1
[2]   C← 2.515517 0.802853 0.010328
[3]   D← 1.432788 0.189269 0.001308
[4]   P←((A←(P≤0.5))×P)+((P>0.5)×(1-P))
[5]   B←(⍟P* 2)*0.5
[6]   Z←((2×A)-1)×-B-((B∘.* 0 1 2)+.×C)÷(1+((B∘.* 1 2 3)+.×D))
[7]   →0
[8]  L1:⎕←'THERE IS NO QUANTILE FOR P = ',⍕A/P
      ∇
```

The following programs were written by the author for the data transformation and data analysis portions of this thesis.

C. NORMALIZATION OF CUMULATIVE FREQUENCIES

```
∇NORM[⎕]∇
∇ NORM;MTX;NOR;ROWAV;GRAV;S;AA;B;AAI;II
  ⎕←'INPUT THE CUMULATIVE FREQUENCIES'
  ⎕←'INSURE THAT THERE ARE NO VALUES OF'
  ⎕←'ZERO (0)  OR OF ONE (1)'
  MTX←⎕
  NOR←NQUAN MTX
  S←ρMTX
  ROWAV←(+/NOR)÷(S[2])
  ROWAV←(S[1],1)ρROWAV
  COLAV←(+⌿NOR)÷(S[1])
  GRAV←(+/(+/NOR))÷((S[1])×(S[2]))
  ⎕←'NORMALIZED VALUES                ROW AVERAGE'
  ⎕←'-----------------------------------------------------'
  ⎕←NOR,ROWAV
  ⎕←' '
  ⎕←'   COLUMN AVERAGES '
  ⎕←'------------------------------'
  ⎕←COLAV
  ⎕←' '
  ⎕←'GRAND  AVERAGE'
  ⎕←'------------------'
  ⎕←GRAV
  ⎕←' '
  AAI←Sρ0
  II←0
 L2:II←II+1
  AA←(NOR[;(II)]-,ROWAV)*2
  AAI[;II]←AA
  →(II<S[2])/L2
  AAI←(S)ρAAI
  AAI←+/AAI
  B←+/((COLAV-GRAV)*2)
  SQR←(B÷AAI)*0.5
  SQR←((S[1]),1)ρSQR
  SSI←GRAV-(ROWAV×SQR)
  AAI←(S[1],1)ρAAI
  GRAV←(S[1],1)ρGRAV
  ⎕←'   B   '
  ⎕←'--------'
  ⎕←B
  ⎕←' '
  ⎕←'    AI'
  ⎕←'----------'
  ⎕←AAI
  ⎕←' '
  ⎕←'SCALE VALUES  = GRAND AVERAGE - (ROW AVERAGE×(B÷AI)*.5)'
  ⎕←'--------------------------------------------------------------'
  ⎕←SSI,GRAV,ROWAV,SQR
  ⎕←' '
  ⎕←' COLUMN AVERAGES '
  ⎕←'------------------'
  ⎕←COLAV
∇
```

D. TRANSFORMATION OF DATA

```
∇TRANS[⎕]∇
      ∇ TRANS;UP;LOW;BETA;ALPHA;MX;COLUP;COLLOW;TRCOL
[1]     ⎕←'INPUT THE COLUMN AVERAGES FROM THE NORM FUNCTION'
[2]     COL←⎕
[3]     S←⍴COL
[4]     COLUP←COL[S]
[5]     COLLOW←COL[1]
[6]     ⎕←'INPUT THE VECTOR YOU WANT TRANSFORMED'
[7]     MX←⎕
[8]     ⎕←'INPUT THE UPPER NUMEER YOU WANT WITH THIS GROUP'
[9]     UP←⎕
[10]    ⎕←'INPUT THE LOWER NUMBER YOU WANT WITH THIS GROUP'
[11]    LOW←⎕
[12]    BETA←(UP-LOW)÷(COLUP-COLLOW)
[13]    ALPHA←UP-BETA×COLUP
[14]    TR←ALPHA+BETA×MX
[15]    TRCOL←ALPHA+BETA×COL
[16]    ⎕←'TRANSFORMED COLUMN UPPER BOUNDS ARE NOW'
[17]    ⎕←TRCOL
[18]    ⎕←' '
[19]    ⎕←' TRANSFORMED DATA '
[20]    ⎕←TR
      ∇
```

E. WILCOXON SIGN RANK TEST

```
      ∇WILCOX[⎕]∇
      ∇ WILCOX;X;Y;DIF;ABSDIF;RANK;SIRANK;SHP;SI;SJ;SDY;N;Z1;Z2
[1]   ⎕←' INPUT THE X VALUES '
[2]   X←⎕
[3]   ⎕←' INPUT THE Y VALUES '
[4]   Y←⎕
[5]   DIF←X-Y
[6]   ABSDIF←|DIF
[7]   SHP←⍴X
[8]   RANK←⍋⍋(,ABSDIF)
[9]   RANK←(SHP)⍴RANK
[10]  SIRANK←RANK×(DIF÷ABSDIF)
[11]  ⎕←'  X        Y        DIF       |DIF|    |RANK|    |SIGN RANK|'
[12]  ⎕←X,Y,DIF,ABSDIF,RANK,SIRANK
[13]  TMIN←+/SI←SIRANK×(SIRANK<0)
[14]  TMIN←+/|TMIN
[15]  TPLUS←+/SJ←SIRANK×(SIRANK>0)
[16]  TPLUS←+/TPLUS
[17]  N←SHP[1]
[18]  SDY←((N×(N+1)×((2×N)+1))÷24)*0.5
[19]  ZMIN←((TMIN)-(0.5+((N×(N+1))÷4)))÷SDY
[20]  ZPLUS←((TPLUS)-(0.5+((N×(N+1))÷4)))÷SDY
[21]  Z1←GAUSS ZMIN
[22]  Z2←GAUSS ZPLUS
[23]  ⎕←' '
[24]  ⎕←'T PLUS  = ',(⍕TPLUS),'   T MINUS  = ',(⍕TMIN)
[25]  ⎕←' '
[26]  ⎕←'Z PLUS  = ',(⍕ZPLUS),'    Z MINUS  = ',(⍕ZMIN)
[27]  ⎕←' '
[28]  ⎕←' P-VALUE FROM Z PLUS = ',(⍕Z2)
[29]  ⎕←' '
[30]  ⎕←' P-VALUE FROM Z MINUS = ',(⍕Z1)
      ∇
```

F. CALCULATION OF CUMULATIVE FREQUENCIES

```
        ∇JUDGES[⎕]∇
        ∇ JUDGES;D:N;SHAPE;FREQ;CUMFREQ
  [1]     ⎕←'INPUT THE RAW DATA POINTS '
  [2]     ⎕←'THE NUMBER OF OBSERVATIONS'
  [3]     ⎕←'IN EACH ROW MUST BE THE SAME'
  [4]     D←⎕
  [5]     N←+/D
  [6]     SHAPE←⍴D
  [7]     N←N[1]
  [8]     FREQ←D[;⍳(SHAPE[2])]÷N
  [9]     ⎕←'RELATIVE FREQUENCY'
  [10]    ⎕←'------------------'
  [11]    ⎕←FREQ
  [12]    ⎕←' '
  [13]    CUMFREQ←+\FREQ
  [14]    ⎕←'CUMULATIVE FREQUENCY'
  [15]    ⎕←'--------------------'
  [16]    ⎕←CUMFREQ
  [17]    ⎕←' '
        ∇
```

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